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Using Multistage Laplace Adomian Decomposition Method to Solve Chaotic Financial System

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Abstract

In this paper, a new reliable algorithm, multistage Laplace Adomian decomposition method (MLADM) based on standard Laplace – Adomian method, is presented to solve a time- fractional financial model for both chaotic and non- chaotic. The new algorithm is just a simple modification of Laplace- Adomian method (LAM). This method is considered as an algorithm in a sequence of small intervals for obtaining accurate approximate solutions. The study depicts that the LADM provides reliable results for $t \ll 1$. Numerical comparisons between the MLADM and the classical Runge- Kutta fourth order method (RK4) in the case of integer-order derivatives solutions indicates that the MLADM gives better output with high accuracy and is a promising technique for nonlinear systems of integer and fractional order.

Keywords: Laplace Adomian decomposition method; Runge- Kutta fourth order method; Laplace- Adomian method; Riemann Liouville integral; Chaos theory.

1 Introduction

For the past four decades, a phenomenon called chaos has been investigated by many researchers including mathematicians, engineers and others [1,2]. The first classical chaotic system was discovered in 1963 by Lorenz when he studied atmospheric convection [3]. In 1984, Chau introduced the first chaotic circuit which connects the chaos theory and the nonlinear circuit theory. In 1999, a dual system of Lorenz system was identified by Chen and Ueta through a technique called chaotification [4]. In 2002, a new chaotic system was identified and named as Lü system [5]. Lü et al. [5] put together all the three systems described above into

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one chaotic called unified chaotic system. Chao which theory deals with behaviour of dynamical systems has insightful effects on the numerical solutions based on its initial conditions [6,7]. This is sometimes sensitive to time step sizes with regard to its numerical solutions. It has been identified that the majority real life phenomena and some scientific problem could be modeled by chaotic systems of ordinary differential equations (ODEs). For instance, Chaos theory has been employed in many fields of endeavour such as population dynamics, mathematics, biology, computer science, economics engineering, finance, meteorology and economics [8,9]. Interestingly, many chaotic systems cannot be solved analytically via their complexities. It is, therefore, prudent to explore the power of numerical methods to obtain approximation solutions of such problems. Consider a financial system [10,11]. The system comprise three state variables as

$$\begin{aligned} \frac{dx}{dt} &= z + (y-d)x \\ \frac{dy}{dt} &= 1 - ey - x^2 \\ \frac{dz}{dt} &= -x - fz \end{aligned} \tag{1.1}$$

where x, y *and* z are the state variables d, e and f are positive real numbers. Here d denotes the amount of saving, e represents the cost per investment, and the commercial markets elasticity demand is denoted by f , $i = 1, 2, 3,$ are parameters dealing with the order of the fractional time derivative in the Caputo perspective . Clearly, the classical integer-order financial can be seen as exceptional situation from fractional –order system by putting $\alpha_1 = \alpha_2 = \alpha_3 = 1$ where the chaotic occurs when $d = 3, e = 0.1$ and $f = 1$. We obtain the following system of fractional time derivatives equation;

$$\begin{aligned} D_*^{\alpha_1} x(t) &= z + (y-d)x \\ D_*^{\alpha_2} y(t) &= 1 - ey - x^2 \\ D_*^{\alpha_3} z(t) &= -x - fz \end{aligned} \tag{1.2}$$

Li and Chen, [12] examined the dynamics of Rössler equation in fractional-order generalizations. They observed in their studies that chaotic behaviour exists as low as 2.4 in Rössler equation. Moaddy et al., [13] studied fractional order dynamics behaviour Rössler chaotic using the nonstandard finite difference scheme. The study shows that smallest value that chaotic occurs is 2.1

In recent times, Roslan et al. [14] explored Euler's method to obtain solution to chaotic system. Even though the results obtained led to butterfly- shape but cannot provide good accuracy. One of the common and widely used numerical methods is RK4 for simulating the solution of chaotic system [12,15,16] and has been mostly used as comparison method [17-20]. There are other powerful methods of solving chaotic systems such as Laplace Adomian decomposition method (LADM) [21,22]. This method has been applied in solving different kinds of differential equations. Unfortunately, it has some setbacks including most semi- analytic scheme. By using the LADM, we obtain a series solution which is, essentially a truncated series solution. This obtained solution does not reveal the real characteristics of the given problem but provides a good approximation to the exact solution in a small area. By the fact that the LADM leads to small convergence area, a multi-staging technique referred to as the Multistage Laplace Adomian Decomposition Method (MLADM) is proposed.

In this paper we intend to apply the LADM and MLADM to solve financial system [10,11], using the concept of fractional calculus for both chaotic and non-chaotic scenarios and compare to fourth order Runge-Kutta method.

2 Basic Definitions and Notations

This section deals with a number of basic definitions and notations of fractional calculus that will help in the subsequent sections [9].

Definition 1. A function $f(x)$ having the positive values of x is recognized in the space D_α $(\alpha \in \mathrm{R})$ if it is expressed in the form $f(x) = x^q f_1(x)$ and for some $\mathrm{q} > \alpha$, where $f_1(x)$ is continuous in $[0, \infty)$, and it is known to be in the space D_α^n if $f^{(n)} \in D_{n,} n \in N$.

Definition 2. The Riemann Liouville integral operator of a given order $\alpha > 0$ with $e \geq 0$ is expressed as

$$\left(J_a^\alpha p\right)(x) = \frac{1}{\Gamma(\alpha)} \int_e^x (x-t)^{n-1} f(t) dt, \quad x > \alpha, \qquad (2.1)$$
$$\left(J_e^0 f\right)(x) = f(x),$$

For the properties of the operator see [23], we need only the following. For $p \in B_n, \alpha > 0, \beta > 0$, $c \in R$ and γ >-1, one gets

$$\left(J_e^\alpha J_e^\beta f\right)(x) = \left(J_e^\beta J_e^\alpha f\right)(x) = \left(J_e^{\alpha+\beta} f\right)(x), \qquad (2.2)$$
$$J_e^\alpha x^\gamma = \frac{x^{\gamma+\alpha}}{\Gamma(e)} \beta_{(x-e)/x}(\alpha, \gamma+1),$$

where $\beta_\tau(\alpha, \gamma+1)$ characterizes the incomplete beta function stated as

$$D_\tau(\alpha, \gamma+1) = \int_0^\tau t^{\alpha_1} (1-t)^\gamma dt,$$

$$J_e^\alpha f^{cx} = f^{cx} (x-e)^\alpha \sum_{k=0}^{\infty} \frac{[c(c-e)]^k}{\Gamma(\alpha+k+1)}. \qquad (2.3)$$

The Riemann Liouville derivative has some challenges when applied to real life situations with fractional differential equations. Thus, at this point we explore a modified version of fractional differential operator D_e^α which has been used in Caputo work on the theory of viscoelasticity.

Definition 3. The Caputo fractional derivative of $p(x)$ of order $\alpha > 0$ with $a \geq 0$ is expressed as

$$\left(D_e^\alpha f\right)(x) = \left(J_e^{m-\alpha} f^{(m)}\right)(x) = \frac{1}{\Gamma(m-\alpha)} \int_e^x \frac{f^{(m)}(t)}{(x-t)^{(\alpha+1-m)}} dt, \qquad (2.4)$$

for $m-1<\alpha\leq m,\ m\in N,\ x\geq e,\ f(x)\ \in \mathrm{D}^m_{-1}$. Many researchers employed the Caputos fractional order derivatives for $m-1<\alpha\leq m,\ \ f(x)\ \in \mathrm{D}^m_{\alpha}$, and $\alpha\geq -1$, one obtains

$$\left(J_e^{\alpha} D_e^{\alpha} f\right)(x) = J^{(m)} D^{(m)} p(x) = f(x) - \sum_{k=0}^{m-1} f^{k}(e) \frac{(x-e)^k}{k!}, \tag{2.5}$$

3 Laplace Adomian Decomposition Method (LADM)

In this section, we present a Laplace Adomian decomposition method for obtaining a solution of differential equations expressed operator form:

$$L_t u + S(u) + M(u) = f \tag{3.1}$$

with initial condition

$$u(x,0) = g(x) \tag{3.2}$$

Where L_t denotes a first-order differential operator, P represents a linear operator, M denotes a non-linear operator and f provides the source term. We initially explore and apply Laplace transform to both faces of equation (3.1) and then substitute the initial condition (3.2).

$$\begin{aligned} &L[L_t u] + L[P(u)] + L[M(u)] = L[f] \\ &sL[u] - g(x) = L[f] - L[P(u)] - L[M(u)] \end{aligned} \tag{3.3}$$

$$L[u] = \frac{g(x)}{s} + \frac{L[f]}{s} - \frac{L[P(u)]}{s} - \frac{L[M(u)]}{s} \tag{3.4}$$

The infinite series in terms of LADM solution $u(x,t)$ is defined as

$$u(x,t) = \sum\nolimits_{m=0}^{\infty} u_m \tag{3.5}$$

The Adomian polynomials technique is used to write the non-linear term which is expressed as [24]:

$$M(u) = \sum\nolimits_{m=0}^{\infty} A_m \tag{3.6}$$

$$A_m = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m}[M\sum\nolimits_{i=0}^{\infty} \lambda^i u_i]\right] \tag{3.7}$$

By substituting (3.5) and (3.6) into (3.3) leads to:

$$L[\sum\nolimits_{m=0}^{\infty} u_m] = \frac{g(x)}{s} + \frac{L[f]}{s} - \frac{L[P(u)]}{s} - \frac{L[\sum\nolimits_{m=0}^{\infty} A_m]}{s} \tag{3.8}$$

Following, (3.8), the following recursive formula can be stated:

$$L[u_0] = \frac{g(x)}{s} + \frac{L[(f)]}{s} \tag{3.9}$$

$$L[u_{n+1}] = -\frac{L[P(u)]}{s} - \frac{L[A_n]}{s} \tag{3.10}$$

By carefully taking the inverse Laplace transform to both ends of (3.9) and (3.10) we get $u_m (m \geq 0)$ subsequently substituted into (3.5).

3.1 Multistage Laplace Adomian Decomposition Method (MLADM)

A robust and efficient method is required to provide approximate solutions to system of differential equations for large t $(t >> 0)$ both linear and non-linear, multi- staging the solution approach is one of the methods.

The interval with which the differential equation (1) solution is to be determined is made to be $[0,T]$. The solution $[0,T]$ is further divided in M subintervals $(m = 1, 2, ..., M)$ given equal step size $h = T / M$ having the interval at end points $t_m = mh$.

The LADM scheme is applied initially to determine the approximate solutions x, y and z of (1), over the interval $[0, t_1]$ by employing the initial condition $x(0), y(0)$ and $z(0)$ in that order. In order to determine the approximate solution of (1) with respect to the next interval $[t_1, t_2]$, the $x(t_1), y(t_1)$ and $z(t_1)$ are considered as the new initial condition. In general terms the iterative scheme is continued for any m with the right endpoints $x(t_{n-1}), y(t_{n-1})$ and $z(t_{n-1})$ at the previous interval being considered as the new initial condition for the interval $[t_{n-1}, t_n]$.

4 Application

In this section, we explore and employ the Laplace Adomian decomposition method to financial system (1). The basic procedure of Laplace Adomian decomposition method is employed to the fractional financial model and is expressed as

$$\begin{aligned} \mathcal{L}[D^{\alpha_1}x] &= \mathcal{L}[\mathrm{z}] + \mathcal{L}[\mathrm{xy}] - d\mathcal{L}[\mathrm{x}] \\ \mathcal{L}[D^{\alpha_2}y] &= \mathcal{L}[1] - e\,\mathcal{L}[\mathrm{y}] - \mathcal{L}[\mathrm{x}^2] \\ \mathcal{L}[D^{\alpha_3}z] &= -\mathcal{L}[\mathrm{x}] - f\mathcal{L}[\mathrm{z}] \end{aligned} \tag{4.1}$$

$$s^{\alpha_1}\mathcal{L}[\mathrm{x}] - s^{\alpha_1-1}\,x(0) = \mathcal{L}[\mathrm{z}] + \mathcal{L}[\mathrm{xy}] - d\mathcal{L}[\mathrm{x}]$$

$$s^{\alpha_2}\mathcal{L}[\mathrm{y}] - s^{\alpha_2-1}\,y(0) = \mathcal{L}[1] - e\,\mathcal{L}[\mathrm{y}] - \mathcal{L}[\mathrm{x}^2]$$

$$s^{\alpha_3}\mathcal{L}[\mathrm{z}] - s^{\alpha_3-1}\,z(0) = -\mathcal{L}[\mathrm{x}] - f\mathcal{L}[\mathrm{z}]$$

$$\mathcal{L}[\mathrm{x}] = \frac{x(0)}{s} + \frac{1}{s^{\alpha_1}}\mathcal{L}[\mathrm{z}] + \frac{1}{s^{\alpha_1}}\mathcal{L}[\mathrm{xy}] - \frac{\mathrm{d}}{s^{\alpha_1}}\mathcal{L}[\mathrm{x}]$$

$$\mathcal{L}[\mathrm{y}] = \frac{y(0)}{s} + \frac{1}{s^{\alpha_2}}\mathcal{L}[1] - \frac{e}{s^{\alpha_2}}\mathcal{L}[\mathrm{y}] - \frac{1}{s^{\alpha_2}}\mathcal{L}[\mathrm{x}^2] \quad (4.2)$$

$$\mathcal{L}[\mathrm{z}] = \frac{z(0)}{\mathrm{s}} - \frac{1}{s^{\alpha_3}}\mathcal{L}[\mathrm{x}] - \frac{f}{s^{\alpha_3}}\mathcal{L}[\mathrm{z}]$$

The solution of the financial system takes the form

$$x(t) = \sum_n^\infty X_n \qquad y(t) = \sum_n^\infty Y_n \qquad z(t) = \sum_n^\infty Z_n$$

The non-linear Adomian polynomials are expressed as:

$$\mathrm{xy} = \sum_n^\infty A_n \qquad \mathrm{x}^2 = \sum_n^\infty B_n$$

Then (4.2) can be written as a recursive formula in parameterized form as:

$$\mathcal{L}\left[\sum \lambda^n X_n\right] = \frac{x(0)}{s} + \frac{\lambda}{s^{\alpha_1}}\mathcal{L}\left[\sum \lambda^n Z_n\right] + \frac{\lambda}{s^{\alpha_1}}\mathcal{L}\left[\sum \lambda^n A_n\right] - \frac{\mathrm{d}\lambda}{s^{\alpha_1}}\mathcal{L}\left[\sum \lambda^n X_n\right]$$

$$\mathcal{L}\left[\sum \lambda^n Y_n\right] = \frac{y(0)}{s} + \frac{\lambda}{s^{\alpha_2}}\mathcal{L}[1] - \frac{e\lambda}{s^{\alpha_2}}\mathcal{L}\left[\sum \lambda^n Y_n\right] - \frac{\lambda}{s^{\alpha_2}}\mathcal{L}\left[\sum \lambda^n B_n\right]$$

$$\mathcal{L}[\sum \lambda^n Z_n] = \frac{z(0)}{\mathrm{s}} - \frac{\lambda}{s^{\alpha_3}}\mathcal{L}[\sum \lambda^n X_n] - \frac{f\lambda}{s^{\alpha_3}}\mathcal{L}[\sum \lambda^n Z_n] \quad (4.3)$$

Comparing equal powers of λ in equation (4.3), we have:

$$\mathcal{L}[X_0] = \frac{x(0)}{s} \qquad \mathcal{L}[Y_0] = \frac{y(0)}{s} + \frac{1}{s^{\alpha_2+1}} \qquad \mathcal{L}[Z_0] = \frac{z(0)}{\mathrm{s}}$$

$$X_0 = \mathcal{L}^{-1}\left[\frac{x(0)}{s}\right] \qquad Y_0 = \mathcal{L}^{-1}\left[\frac{y(0)}{s} + \frac{1}{s^{\alpha_2+1}}\right] \qquad Z_0 = \mathcal{L}^{-1}\left[\frac{z(0)}{s}\right] \quad (4.4)$$

$$\mathcal{L}[X_{n+1}] = \frac{1}{s^{\alpha_1}}\mathcal{L}[Z_n] + \frac{1}{s^{\alpha_1}}\mathcal{L}[A_n] - \frac{\mathrm{d}}{s^{\alpha_1}}\mathcal{L}[X_n]$$

$$\mathcal{L}[Y_{n+1}] = -\frac{e}{s^{\alpha_2}}\mathcal{L}[Y_n] - \frac{1}{s^{\alpha_2}}\mathcal{L}[B_n]$$

$$\mathcal{L}[Z_{n+1}] = -\frac{1}{s^{\alpha_3}}\mathcal{L}[X_n] - \frac{f}{s^{\alpha_3}}\mathcal{L}[Z_n]$$

$$X_{n+1} = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha_1}}\mathcal{L}[Z_n] + \frac{1}{s^{\alpha_1}}\mathcal{L}[A_n] - \frac{\mathrm{d}}{s^{\alpha_1}}\mathcal{L}[X_n]\right] \quad (4.5)$$

$$Y_{n+1} = \mathcal{L}^{-1}\left[-\frac{e}{s^{\alpha_2}}\mathcal{L}[Y_n] - \frac{1}{s^{\alpha_2}}\mathcal{L}[B_n]\right]$$

$$Z_{n+1} = \mathcal{L}^{-1}\left[-\frac{1}{s^{\alpha_3}}\mathcal{L}[X_n] - \frac{f}{s^{\alpha_3}}\mathcal{L}[Z_n]\right]$$

The system is solved with the initial condition $x(0) = 2,\ y(0) = 3$ and $z(0) = 2$. For $d = 3,\ e = 0.1$ and $f = 1$ we have a chaotic system and $d = 2,\ e = -0.1$ and $f = 1.6$ correspond to a non-chaotic system. The recursive relations (4.3) and (4.5) are evaluated with the aid of Mathematica 10.0 version to obtain the solution up to a 10 terms approximation for the time range [0, 300] with a time step size 0.01. MLADM is implemented by dividing the solution interval $[0, 300]$

5 Results and Discussion

The chaotic case of the classical finance system was obtained by applying the LADM and MLADM which is then implemented in Mathematica 10.0 version, the 21 terms approximate LADM solution was arrived at. The first few terms of the recursive relation (4.5) are obtained as

$$X_0 = 2$$

$$Y_0 = \frac{3 + t^{\alpha_2}}{\Gamma(1+\alpha_2)}$$

$$Z_0 = 2$$

$$X_1 = 2t^{\alpha_1}\left(\frac{1}{\Gamma(1+\alpha_1)} + \frac{t^{\alpha_2}}{\Gamma(1+\alpha_1+\alpha_2)}\right)$$

$$Y_1 = t^{\alpha_2}\left(-\frac{4.3}{\Gamma(1+\alpha_2)} - \frac{0.1\,t^{\alpha_2}}{\Gamma(1+2\alpha_2)}\right)$$

$$Z_1 = \frac{-4t^{\alpha_3}}{\Gamma(1+\alpha_3)}$$

$$X_2 = t^{\alpha_1}\left[\frac{2\,t^{\alpha_1+\alpha_2}\,\Gamma(1+\alpha_1+\alpha_2)}{\Gamma(1+\alpha_1)\,\Gamma(1+\alpha_2)\,\Gamma(1+2\alpha_1+\alpha_2)} - \frac{0.2\,t^{2\alpha_2}}{\Gamma(1+\alpha_1+2\alpha_2)} + t^{\alpha_2}\left(\frac{-8.6 + \frac{2\,t^{\alpha_1+\alpha_2}\Gamma(1+\alpha_1+2\alpha_2)}{\Gamma(1+\alpha_2)\,\Gamma(1+2\alpha_1+2\alpha_2)}}{\Gamma(1+\alpha_1+\alpha_2)}\right) - \frac{4t^{\alpha_3}}{\Gamma(1+\alpha_1+\alpha_3)}\right]$$

$$Y_2 = t^{\alpha_2}\left[-\frac{8t^{\alpha_1}}{\Gamma(1+\alpha_1+\alpha_2)} + t^{\alpha_2}\left(\frac{0.43}{\Gamma(1+2\alpha_2)} - \frac{8t^{\alpha_1}}{\Gamma(1+\alpha_1+2\alpha_2)} + \frac{0.01\,t^{\alpha_2}}{\Gamma(1+3\alpha_2)}\right)\right]$$

$$Y_2 = t^{\alpha_2}\left[-\frac{8t^{\alpha_1}}{\Gamma(1+\alpha_1+\alpha_2)} + t^{\alpha_2}\left(\frac{0.43}{\Gamma(1+2\alpha_2)} - \frac{8t^{\alpha_1}}{\Gamma(1+\alpha_1+2\alpha_2)} + \frac{0.01\,t^{\alpha_2}}{\Gamma(1+3\alpha_2)}\right)\right]$$

$$Z_2 = 2t^{\alpha_3}\left[-\frac{t^{\alpha_1}}{\Gamma(1+\alpha_1+\alpha_3)} - \frac{t^{\alpha_1+\alpha_2}}{\Gamma(1+\alpha_1+\alpha_2+\alpha_3)} + \frac{2t^{\alpha_3}}{\Gamma(1+2\alpha_3)}\right]$$

into 3000 subintervals $(n = 1, 2, \dots \dots \dots, 3000)$ of equal step size given by $h = 0.1$

For the chaotic case $d = 3$, $e = 0.1$ and $f = 1$, we have the series solution as:

$$x = 2 + 2t - 5.3t^2 - 4.42333t^3 + 5.74392t^4 + 13.3092t^5 - 4.82205t^6 - 28.4528t^7 - 5.57176t^8 + 52.3603t^9 + 38.865t^{10} + 96.7929t^{11} + 4.95108t^{12} - 11.4009t^{13} - 1.1977t^{14} + 0.175363t^{15} + 0.019144t^{16} - 0.000347514t^{17} - 0.0000423286t^{18} - 1.74675e - 10t^{19} + 5.38229e - 10t^{20}$$

$$y = 3 - 3.3t - 3.835t^2 + 5.86117t^3 + 9.5768t^4 - 6.866t^5 - 20.4022t^6 + 1.34447t^7 + 40.6071t^8 + 18.4081t^9 - 68.6147t^{10} + 14.9803t^{11} + 44.9862t^{12} + 4.17385t^{13} - 1.75135t^{14} - 0.188676t^{15} + 0.0104597t^{16} + 0.00115873t^{17} - 3.16501e - 6t^{18} - 5.80154e - 7t^{19}$$

$$z = 2 - 4t + t^2 + 1.43333t^3 + 0.7475t^4 - 1.29828t^5 - 2.00182t^6 + 0.974838t^7 + 3.43474t^8 + 0.237447t^9 - 5.25977t^{10} - 0.483038t^{11} + 2.04308t^{12} + 0.258554t^{13} - 0.0481953t^{14} - 0.00576637t^{15} + 0.000138421t^{16} + 0.0000174021t^{17} + 2.7936e - 10t^{18} - 5.66557e - 10t^{19}$$

The effectiveness and accuracy of the LADM and MLADM is studied by comparing their solutions to the RK4 solution for the parameter where the system is chaotic with the initial conditions $x(0) = 2, y(0) = 3$ and $z(0) = 2$. The RK4 with time steps $\Delta t = 0.01$ with the number of significant digits set to 16 is used. Table 1 presents the absolute errors between the 21-term LADM solutions and the 21-term MLADM solutions for $d = 3, e = 0.1$, $f = 1$ and the RK4 solutions.

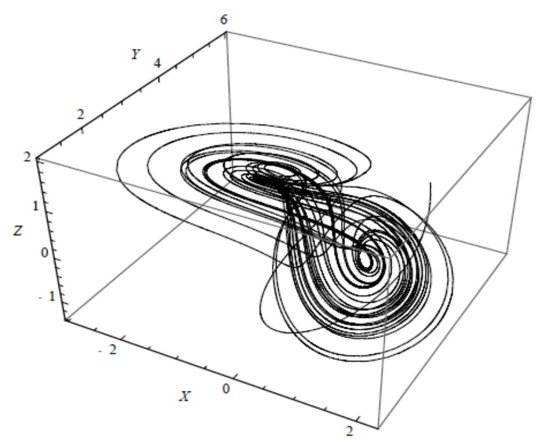

Fig. 1. X-Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$, $e = 0.1$ and $f = 1$

In Table 1, we can see that the LADM only provides valid result for $t << 1$. The MLADM solutions on the time step $\Delta t = 0.01$ for the chaotic case matches with the RK4 solutions on the time step $\Delta t = 0.01$ to at least 4 decimal places. Therefore, for the classical finance chaotic system we notice that the MLADM solutions matches with the RK4 solutions to a significant degree. The $z + (y - d)x$, $1 - ey - x^2$ and $-x - fz$ phase portraits for the non-chaotic case is determined employing the 21-term MLADM solutions are depicted in Fig. 1 to Fig. 4. In Figs. 5, 6 and 7 we show the time series plots of the model equations respectively. These graphs depict oscillatory variations in the time interval of $0 \le t \le 30$ seconds using MLADM when $d = 3$, e=0.1 and $f = 1$.

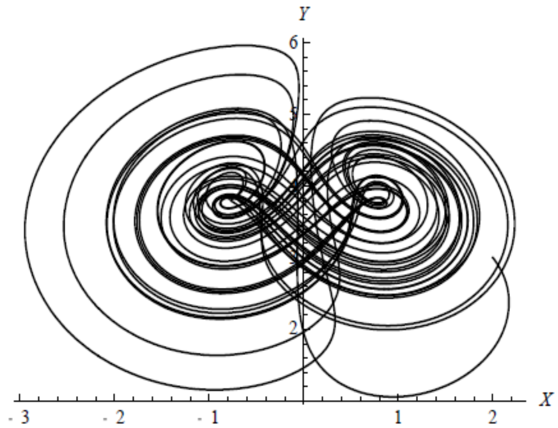

Fig. 2. X-Y Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$, $e = 0.1$ and $f = 1$, $\alpha = 1$

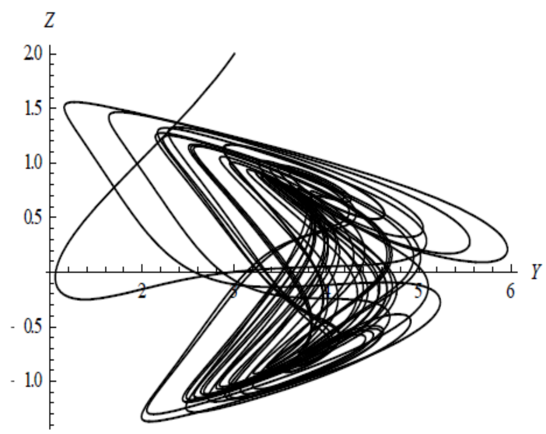

Fig. 3. Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$, $e = 0.1$ and $f = 1$, $\alpha = 1$

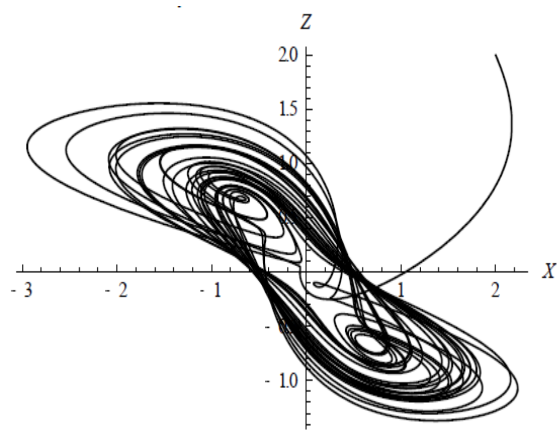

Fig. 4. X-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 3$, $e = 0.1$ and $f = 1$, $\alpha = 1$

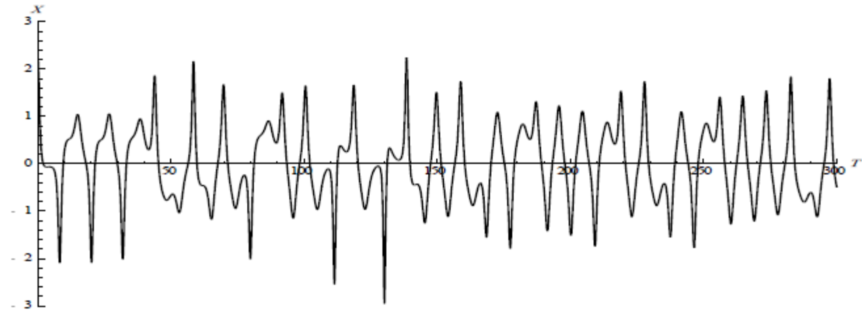

Fig. 5. X(t) time series using 10-term MLADM for $d = 3$, $e = 0.1$ and $f = 1$

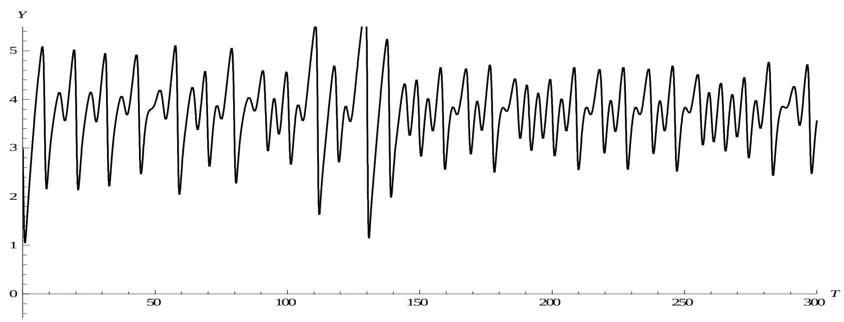

Fig. 6. Y(t) time series using 10-term MLADM for $d = 3$, $e = 0.1$ and $f = 1$

Table 1. Absolute differences between 10-term LADM and 10-term MLADM with RK4 solutions ($\Delta t = 0.01$) for $d = 3$, $e = 0.1$ and $f = 1$

t	$\lvert LADM_{0.01} - RK4_{0.01}\rvert$			$\lvert MLADM_{0.01} - RK4_{0.01}\rvert$		
	Δx	Δy	Δz	Δx	Δy	Δz
0.15	4.623E-10	2.812E-11	3.594E-09	4.623E-10	2.812E-11	3.594E-09
4.43	8.647E-05	1.801E-05	6.452E-04	1.008E-09	1.297E-09	1.512E-09
22.04	1.066E+00	1.166E+00	4.594E+00	4.494E-09	8.654E-09	7.228E-10
46.71	5.340E+02	2.479E+03	5.249E+03	4.254E-10	7.486E-09	1.570E-09
71.13	1.972E+05	2.159E+06	3.763E+06	5.482E-09	2.697E-08	9.049E-09
95.52	3.741E+07	6.570E+08	1.016E+09	2.785E-10	2.058E-08	6.010E-11
120.04	3.169E+09	7.231E+10	1.046E+11	4.052E-10	1.922E-08	1.051E-10
156.33	9.462E+10	2.484E+12	3.468E+12	3.465E-09	1.601E-08	4.111E-09
185.51	2.147E+12	6.196E+13	8.447E+13	2.261E-09	1.567E-08	4.535E-09
212.12	9.249E+12	2.767E+14	3.738E+14	1.928E-09	1.130E-08	2.424E-09
278.23	4.314E+13	1.333E+15	1.787E+15	6.767E-10	1.211E-08	2.196E-10

For the non-chaotic case $d = 2$, $e = -0.1$ and $f = 1.6$, we have the series solution as:

$$x = 2 + 4t - 3.3t^2 - 9.40333t^3 - 8.87358t^4 + 14.5722t^5 + 37.4136t^6 + 13.0765t^7 - 70.9391t^8 - 119.882t^9 + 17.3112t^{10} - 158.999t^{11} - 89.8305t^{12} - 4.81831t^{13} + 2.84324t^{14} + 0.383951t^{15} + 0.00429468t^{16} - 0.0012362t^{17} - 0.0000420489t^{18} + 1.70486e - 8t^{19} + 5.38229e - 10t^{20}$$

$$y = 3 - 2.7t - 8.135t^2 - 1.2045t^3 + 15.9732t^4 + 20.2857t^5 - 7.88893t^6 - 59.1442t^7 - 53.5294t^8 + 68.4484t^9 + 210.243t^{10} + 154.278t^{11} - 25.6593t^{12} - 22.3443t^{13} - 2.38427t^{14} + 0.145652t^{15} + 0.0302603t^{16} + 0.00928876t^{17} - 0.0000171922t^{18} - 5.80154e - 7t^{19}$$

$$z = 2 - 5.2t + 2.16t^2 - 0.052t^3 + 2.37163t^4 + 1.01579t^5 - 2.69958t^6 - 4.72776t^7 - 0.689006t^8 + 8.00462t^9 + 10.7075t^{10} + 10.8071t^{11} + 0.721589t^{12} - 0.652907t^{13} - 0.0998574t^{14} - 0.000655983t^{15} + 0.00047164t^{16} + 0.0000171717t^{17} - 1.58464e - 8t^{18} - 5.66557e - 10t^{19}$$

Fig. 7. Z(t) time series using 10-term MLADM for $d = 3$, $e = 0.1$ and $f = 1$

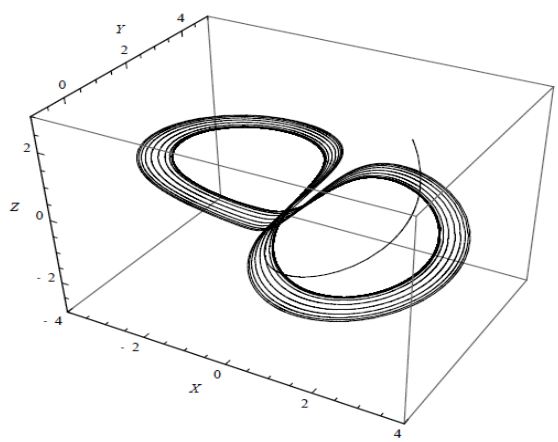

Fig. 8. X-Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$ and $f = 1.6$, $\alpha = 1$

In Table 2, we can also notice that the LADM only provides valid result for $t << 1$. The MLADM solutions for the non-chaotic case is in agreement with the RK4 solutions to at least 4 decimal places. The $z + (y - d)x$, $1 - ey - x^2$ and $-x - fz$ depict the phase portraits for the non-chaotic classical finance system obtained employing the 21-term MLADM solutions and are shown in Fig. 8 to Fig. 11.

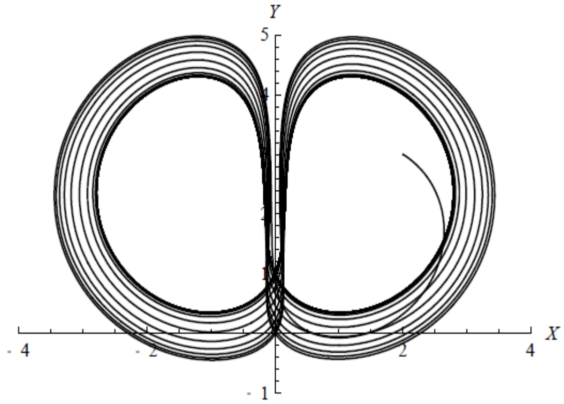

Fig. 9. X-Y Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$ and $f = 1.6$, $\alpha = 1$

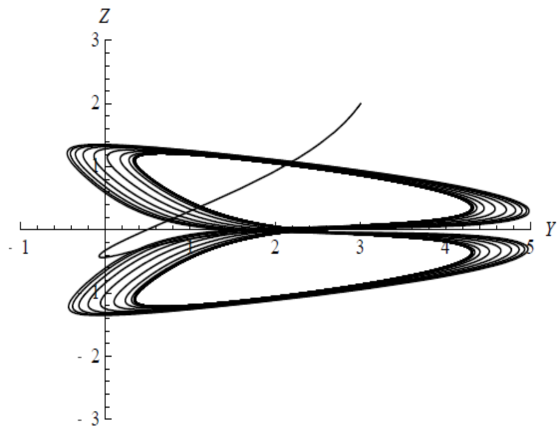

Fig. 10. Y-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$ and $f = 1.6$, $\alpha = 1$

Table 2. Absolute differences between 10-term LADM and 10-term MLADM with RK4 solutions ($\Delta t = 0.01$) for $d = 2$, $e = -0.1$ and $f = 1.6$

	$\lvert LADM_{0.01} - RK4_{0.01} \rvert$			$\lvert MLADM_{0.01} - RK4_{0.01} \rvert$		
t	Δx	Δy	Δz	Δx	Δy	Δz
0.15	2.407E-10	4.552E-09	6.828E-10	2.407E-10	4.552E-09	6.828E-10
4.43	4.003E-04	1.049E-03	8.516E-05	4.064E-10	3.177E-10	4.378E-10
22.04	1.067E+01	2.777E+00	5.612E-01	7.022E-11	5.363E-10	3.718E-10
46.71	5.788E+03	6.718E+03	4.383E+02	2.435E-10	4.909E-11	4.626E-10
71.13	2.885E+06	4.328E+06	1.786E+05	2.417E-11	1.541E-11	1.839E-11
95.52	6.804E+08	1.110E+09	2.574E+07	1.224E-09	1.553E-09	9.840E-10
120.04	6.613E+10	1.117E+11	1.381E+09	4.385E-10	1.947E-11	1.583E-10
156.33	2.136E+12	3.668E+12	2.377E+10	8.462E-10	7.770E-10	6.206E-10
185.51	5.118E+13	8.884E+13	2.212E+11	3.171E-11	7.643E-11	3.571E-11
212.12	2.252E+14	3.924E+14	3.903E+11	6.414E-12	9.170E-12	3.612E-11
278.23	1.071E+15	1.872E+15	6.907E+11	1.762E-06	8.250E-06	1.779E-08

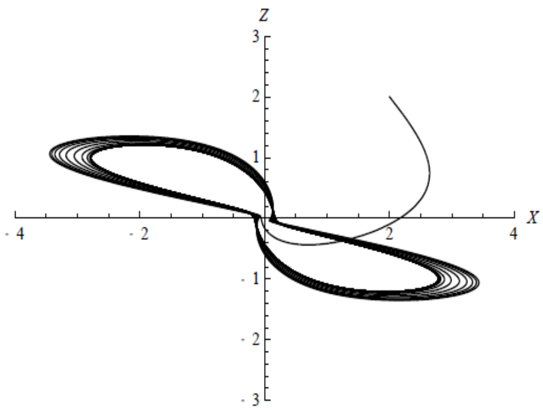

Fig. 11. X-Z Phase portrait using 10-term MLADM on $\Delta t = 0.01$ for $d = 2$, $e = -0.1$ and $f = 1.6$, $\alpha = 1$

6 Conclusion

In this work, a new numerical technique to deal with time- fractional classical financial model is proposed. The method is a only modification of the standard laplace-Adomaian method. Comparisons were made among the LADM, MLADM and the fourth-order Runge- Kutta (RK4) method. Following the numerical results obtained in the case of chaotic and non-chaotic the MLADM and RK4 were consistent. However, in the case of chaotic solution the MLADM performed relatively better than that of RK4.

Competing Interests

Authors have declared that no competing interests exist.

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