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On V_4-cordial Labeling of Graphs

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Original Research Article

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Abstract

In this Research article we prove the following results:

1. The crown graph $C_n \bigodot K_1$ is V_4-cordial for all n.
2. The armed crown AC_n is V_4-cordial for all n.
3. The pan graph C_n^{+1} is V_4-cordial for all n.
4. The corona graph $C_n \bigodot mK_1$ is V_4-cordial for all n and m.

Keywords: Klein-four group; V_4-cordial labeling; crown; armed crown; pan graph.

2010 Mathematics Subject Classification: 05C78.

1 Introduction

In this research article, by a graph we mean finite, connected, undirected and simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$. Here we discuss V_4-cordial labeling of standard graphs which are obtained by using some graph operations.

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In this paper we consider the following definitions.

Definition 1.1. The *Corona* $G_1 \bigodot G_2$ of two graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 1.2. The *Crown* $C_n \bigodot K_1$ is obtained by joining a pendant edge to each vertex of cycle C_n.

Definition 1.3. An *Armed Crown* is a graph in which path P_2 is attached at each vertex of cycle C_n by an edge. It is denoted by AC_n where n is the number of vertices in cycle C_n.

Definition 1.4. The *n-pan* graph C_n^{+1} is the graph obtained by joining a cycle graph C_n to a complete graph K_1 with a bridge. In other words the pan graph is obtained by adding a pendant edge to any vertex of cycle C_n.

Definition 1.5. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).
The latest updates of various graph labeling techniques can be found in Gallian [1].

Definition 1.6. Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.

Where

$v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.
We note that if $A =< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

Definition 1.7. Let $V_4 = Z_2 \times Z_2 = \{0 =< 0, 0 >, a =< 1, 0 >, b =< 0, 1 >, c =< 1, 1 >\}$ be the Klein-four group in which the operation $*$ is defined as follows:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

The graph $G = (V(G), E(G))$, with vertex set $V(G)$ and edge set $E(G)$, is said to be V_4-cordial if there exist a mapping $f : V(G) \rightarrow V_4$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$
(i) $|v_f(p) - v_f(q)| \leq 1$; for all $p, q \in V_4$,
(ii) $|e_f(p) - e_f(q)| \leq 1$; for all $p, q \in V_4$.

Where

$v_f(p)$=the number of vertices with label p;
$v_f(q)$=the number of vertices with label q;
$e_f(p)$=the number of edges with label p;
$e_f(q)$=the number of edges with label q.

For any undefined term in graph theory we rely upon Gross and Yellen [2].

The concept of V_4-cordiality was introduced by A. Riskin [3] and proved the following result.

1. Complete graph K_n is V_4-cordial if and only if $n < 4$.
2. The Star graph $K_{1,n}$ is V_4-cordial.

M. Seenivasan and A. Lourdusamy [4] proved the following results.

1. If G is an Eulerian graph with q edges, where $q \equiv 2(mod4)$, then G has no V_4-cordial labeling.
2. If f be a V_4-cordial labeling of a graph G with $p \geq 4$ and uv be an edge of G such that $f(u) = 0$ and $f(u) \neq f(v)$. Then the graph $G^{'}$ obtained from G by replacing the edge uv by a path of length five is V_4-cordial.
3. The Path P_4 and P_5 are not V_4-cordial.
4. All Trees except P_4 and P_5 are V_4-cordial.
5. The Cycle C_n is V_4-cordial if and only if $n \neq 4$ or 5 or $n \not\equiv 2(mod4)$.

O. Pechenik and J. Wise [5] proved the following results.

1. The Complete Bipartite graph $K_{m,n}$ is V_4-cordial if and only if m and n are not both congruent to $2(mod4)$.
2. The Path P_n is V_4-cordial unless $n \in \{4, 5\}$.
3. The Cycle C_n is V_4-cordial if and only if $n \notin \{4, 5\}$ and $n \not\equiv 2(mod4)$.
4. All Ladders $P_2 \times P_k$ are V_4-cordial, except $P_2 \times P_2$.
5. The Prism $P_2 \times C_k$ is V_4-cordial if and only if $k \not\equiv 2(mod4)$.
6. The d-dimensional hypercube Q_d is V_4-cordial, unless $d = 2$.

In the present work there are new results corresponding to V_4-cordial labeling of graphs are investigated.

2 Main Results

Theorem 2.1. The Crown $C_n \bigodot K_1$ is V_4-cordial for all n.

Proof. Let $G = C_n \bigodot K_1$ be the crown graph. Let $v_1, v_2, ..., v_n$ be the vertices of cycle C_n and $v_1^{'}, v_2^{'}, ..., v_n^{'}$ are the pendant vertices of crown $C_n \bigodot K_1$. We note that $|V(G)| = 2n$ and $|E(G)| = 2n$.

To define V_4-cordial labeling $f : V(G) \to V_4$ we consider the following cases.

Case 1: $n \equiv 0, 3(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n,$
$f(v'_i) = 0; \quad i \equiv 2(mod4);$
$f(v'_i) = a; \quad i \equiv 1(mod4);$
$f(v'_i) = b; \quad i \equiv 0(mod4);$
$f(v'_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n.$

Case 2: $n \equiv 1(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-1,$
$f(v_n) = 0;$
$f(v'_i) = 0; \quad i \equiv 2(mod4);$
$f(v'_i) = a; \quad i \equiv 1(mod4);$
$f(v'_i) = b; \quad i \equiv 0(mod4);$
$f(v'_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-1,$
$f(v'_n) = b.$

Case 3: $n \equiv 2(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-2,$
$f(v_{n-1}) = 0;$
$f(v_n) = b;$
$f(v'_i) = 0; \quad i \equiv 2(mod4);$
$f(v'_i) = a; \quad i \equiv 1(mod4);$
$f(v'_i) = b; \quad i \equiv 0(mod4);$
$f(v'_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-1,$
$f(v'_n) = c.$

The *Table 1* shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4-cordial labeling. Hence, the Crown $C_n \bigodot K_1$ is V_4-cordial for all n.
Let $n = 4p + q$, where $p, q \in N \cup \{0\}$.

Table 1

q	Vertex conditions	Edge conditions
0,2	$v_f(0) = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) = e_f(c)$
1	$v_f(0) = v_f(a) + 1 = v_f(b) = v_f(c) + 1$	$e_f(0) = e_f(a) + 1 = e_f(b) = e_f(c) + 1$
3	$v_f(0) + 1 = v_f(a) = v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1 = e_f(b) = e_f(c) + 1$

Illustration 2.2. The crown $C_5 \bigodot K_1$ and its V_4-cordial labeling is shown in *Fig. 1*.

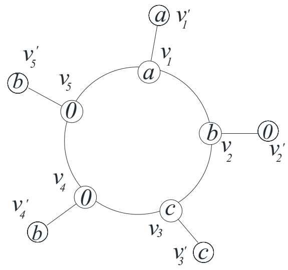

Fig. 1 V_4**-cordial labeling of crown** $C_5 \odot K_1$

Theorem 2.3. The Armed Crown AC_n is V_4-cordial for all n.

Proof. Let $G = AC_n$ be the armed crown. Let $v_1, v_2, ..., v_n$ be the vertices of cycle C_n. Let v_i, v_i' and v_i'' be the vertices of path where $1 \le i \le n$. Here, each v_i' is adjacent to both v_i and v_i'' for all i. We note that $|V(G)| = 3n$ and $|E(G)| = 3n$.

To define V_4-cordial labeling $f : V(G) \to V_4$ we consider the following cases.

Case 1: $n \equiv 0(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \le i \le n,$
$f(v_i') = 0; \quad i \equiv 2(mod4);$
$f(v_i') = a; \quad i \equiv 1(mod4);$
$f(v_i') = b; \quad i \equiv 0(mod4);$
$f(v_i') = c; \quad i \equiv 3(mod4); \quad 1 \le i \le n,$
$f(v_i'') = 0; \quad i \equiv 3(mod4);$
$f(v_i'') = a; \quad i \equiv 1(mod4);$
$f(v_i'') = b; \quad i \equiv 2(mod4);$
$f(v_i'') = c; \quad i \equiv 0(mod4); \quad 1 \le i \le n.$

Case 2: $n \equiv 1(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \le i \le n,$
$f(v_i') = 0; \quad i \equiv 2(mod4);$
$f(v_i') = a; \quad i \equiv 1(mod4);$
$f(v_i') = b; \quad i \equiv 0(mod4);$
$f(v_i') = c; \quad i \equiv 3(mod4); \quad 1 \le i \le n-1,$
$f(v_n') = b;$
$f(v_i'') = 0; \quad i \equiv 3(mod4);$
$f(v_i'') = a; \quad i \equiv 1(mod4);$
$f(v_i'') = b; \quad i \equiv 2(mod4);$
$f(v_i'') = c; \quad i \equiv 0(mod4); \quad 1 \le i \le n-1,$
$f(v_n'') = c.$

Case 3: $n \equiv 2(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$

$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n,$
$f(v'_i) = 0; \quad i \equiv 2(mod4);$
$f(v'_i) = a; \quad i \equiv 1(mod4);$
$f(v'_i) = b; \quad i \equiv 0(mod4);$
$f(v'_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-2,$
$f(v'_{n-1}) = 0;$
$f(v'_n) = c;$
$f(v''_i) = 0; \quad i \equiv 3(mod4);$
$f(v''_i) = a; \quad i \equiv 1(mod4);$
$f(v''_i) = b; \quad i \equiv 2(mod4);$
$f(v''_i) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-2,$
$f(v''_{n-1}) = b;$
$f(v''_n) = c.$

Case 4: $n \equiv 3(mod4)$

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n,$
$f(v'_i) = 0; \quad i \equiv 2(mod4);$
$f(v'_i) = a; \quad i \equiv 1(mod4);$
$f(v'_i) = b; \quad i \equiv 0(mod4);$
$f(v'_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n,$
$f(v''_i) = 0; \quad i \equiv 3(mod4);$
$f(v''_i) = a; \quad i \equiv 1(mod4);$
$f(v''_i) = b; \quad i \equiv 2(mod4);$
$f(v''_i) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-3,$
$f(v''_{n-2}) = 0;$
$f(v''_{n-1}) = c;$
$f(v''_n) = b.$

The *Table 2* shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4-cordial labeling. Hence, the Armed Crown $C_n \bigodot K_1$ is V_4-cordial for all n.

Let $n = 4p + q$, where $p, q \in N \cup \{0\}$.

Table 2

q	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) = e_f(c)$
1	$v_f(0) + 1 = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) + 1 = e_f(c)$
2	$v_f(0) + 1 = v_f(a) + 1 = v_f(b) = v_f(c)$	$e_f(0) + 1 = e_f(a) = e_f(b) + 1 = e_f(c)$
3	$v_f(0) + 1 = v_f(a) + 1 = v_f(b) + 1 = v_f(c)$	$e_f(0) + 1 = e_f(a) = e_f(b) + 1 = e_f(c) + 1$

Illustration 2.4 The armed crown AC_5 and its V_4-cordial labeling is shown in *Fig. 2*.

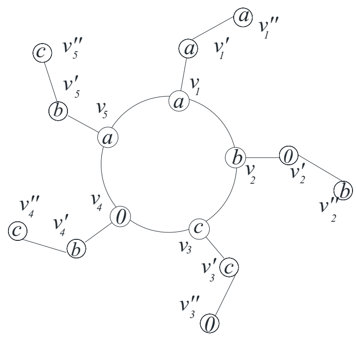

Fig. 2. V_4**-cordial labeling of an armed crown** AC_5

Theorem 2.5. The Pan Graph C_n^{+1} is V_4-cordial for all n.

Proof. Let $G = C_n^{+1}$ be the cycle with one pendant edge known as Pan graph. Let $v_1, v_2, ..., v_n$ be the cycle vertices and $v^{'}$ be the pendant vertex. We note that $|V(G)| = n+1$ and $|E(G)| = n+1$.

To define V_4-cordial labeling $f : V(G) \to V_4$ we consider the following cases.

Case 1: $n \equiv 0(mod8)$.

$f(v_i) = 0; \quad i \equiv 0, 7(mod8);$
$f(v_i) = a; \quad i \equiv 1, 5(mod8);$
$f(v_i) = b; \quad i \equiv 4, 6(mod8);$
$f(v_i) = c; \quad i \equiv 2, 3(mod8); \quad 1 \le i \le n,$
$f(v') = a.$

Case 2: $n \equiv 1(mod8)$.

$f(v_i) = 0; \quad i \equiv 0, 7(mod8);$
$f(v_i) = a; \quad i \equiv 1, 5(mod8);$
$f(v_i) = b; \quad i \equiv 4, 6(mod8);$
$f(v_i) = c; \quad i \equiv 2, 3(mod8); \quad 1 \le i \le n,$
$f(v') = c.$

Case 3: $n \equiv 2(mod8)$.

$f(v_i) = 0; \quad i \equiv 0, 7(mod8);$
$f(v_i) = a; \quad i \equiv 1, 5(mod8);$
$f(v_i) = b; \quad i \equiv 4, 6(mod8);$
$f(v_i) = c; \quad i \equiv 2, 3(mod8); \quad 1 \le i \le n-3,$
$f(v_{n-2}) = a;$
$f(v_{n-1}) = b;$
$f(v_n) = 0;$
$f(v') = 0.$

Case 4: $n \equiv 3(mod 8)$.

$f(v_i) = 0; \quad i \equiv 0, 7(mod8);$
$f(v_i) = a; \quad i \equiv 1, 5(mod8);$
$f(v_i) = b; \quad i \equiv 4, 6(mod8);$
$f(v_i) = c; \quad i \equiv 2, 3(mod8); \quad 1 \leq i \leq n - 5,$
$f(v_{n-4}) = a;$
$f(v_{n-3}) = c;$
$f(v_{n-2}) = b;$
$f(v_{n-1}) = 0;$
$f(v_n) = 0;$
$f(v') = 0.$

Case 5: $n \equiv 4, 5, 7(mod 8)$.

$f(v_i) = 0; \quad i \equiv 0, 7(mod8);$
$f(v_i) = a; \quad i \equiv 1, 5(mod8);$
$f(v_i) = b; \quad i \equiv 4, 6(mod8);$
$f(v_i) = c; \quad i \equiv 2, 3(mod8); \quad 1 \leq i \leq n,$
$f(v') = 0.$

Case 6: $n \equiv 6(mod 8)$.

$f(v_i) = 0; \quad i \equiv 0, 7(mod8);$
$f(v_i) = a; \quad i \equiv 1, 5(mod8);$
$f(v_i) = b; \quad i \equiv 4, 6(mod8);$
$f(v_i) = c; \quad i \equiv 2, 3(mod8); \quad 1 \leq i \leq n - 3,$
$f(v_{n-2}) = 0;$
$f(v_{n-1}) = b;$
$f(v_n) = a;$
$f(v') = 0.$

The *Table 3* shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4-cordial labeling. Hence, the Pan Graph C_n^{+1} is V_4-cordial for all n.

Let $n = 8p + q$, where $p, q \in N \cup \{0\}$.

Table 3

q	Vertex conditions	Edge conditions
0	$v_f(0)+1 = v_f(a) = v_f(b)+1 = v_f(c)+1$	$e_f(0)+1 = e_f(a) = e_f(b)+1 = e_f(c)+1$
1	$v_f(0)+1 = v_f(a) = v_f(b)+1 = v_f(c)$	$e_f(0) = e_f(a)+1 = e_f(b) = e_f(c)+1$
2	$v_f(0) = v_f(a) = v_f(b) = v_f(c)+1$	$e_f(0)+1 = e_f(a) = e_f(b) = e_f(c)$
3,7	$v_f(0) = v_f(a) = v_f(b) = v_f(c)$	$e_f(0) = e_f(a) = e_f(b) = e_f(c)$
4	$v_f(0)+1 = v_f(a)+1 = v_f(b)+1 = v_f(c)$	$e_f(0)+1 = e_f(a)+1 = e_f(b) = e_f(c)+1$
5	$v_f(0)+1 = v_f(a) = v_f(b)+1 = v_f(c)$	$e_f(0) = e_f(a) = e_f(b)+1 = e_f(c)+1$
6	$v_f(0) = v_f(a) = v_f(b)+1 = v_f(c)$	$e_f(0) = e_f(a)+1 = e_f(b) = e_f(c)$

Illustration 2.6 The pan graph C_5^{+1} and its V_4-cordial labeling is shown in *Fig. 3*.

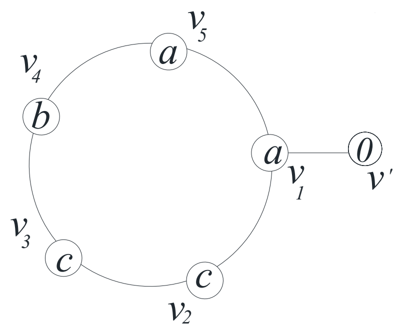

Fig. 3. V_4**-cordial labeling of Pan graph** C_5^{+1}

Theorem 2.7. The Corona $C_n \bigodot mK_1$ of cycle C_n and m complete graph K_1 is V_4-cordial for all n and m.

Proof. Let $G = C_n \bigodot mK_1$ be the corona of cycle C_n and m complete graph K_1. Let $v_1, v_2, ..., v_n$ be the cycle vertices and $v_{11}, v_{12}, ..., v_{1m}, v_{21}, v_{22}, ..., v_{2m}, v_{31}, v_{32}, ..., v_{3m}, ...,$
$v_{i1}, v_{i2}, ..., v_{ij}, ..., v_{im}, ..., v_{n1}, v_{n2}, ..., v_{nj}, ..., v_{nm}$

be the vertices of m complete graph K_1, where $1 \leq i \leq n$ and $1 \leq j \leq m$. We note that $|V(G)| = (m+1)n$ and $|E(G)| = (m+1)n$.

To define V_4-cordial labeling $f : V(G) \to V_4$ we consider the following cases.

Case 1: $n \equiv 0(mod4)$.

Label the vertices $v_1, v_2, ..., v_n$ of cycle as follows.

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n.$

The labeling pattern of the vertices $v_{1j}, v_{2j}, ..., v_{nj}$, where $1 \leq j \leq m$ is divided into following two subcases.

Subcase 1: $m = 1$.

$f(v_{i1}) = 0; \quad i \equiv 2(mod4);$
$f(v_{i1}) = a; \quad i \equiv 1(mod4);$
$f(v_{i1}) = b; \quad i \equiv 0(mod4);$
$f(v_{i1}) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n.$

Subcase 2: $m > 1$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n$ and $2 \leq j \leq m.$

Case 2: $n \equiv 1(mod4)$.

Label the vertices $v_1, v_2, ..., v_n$ of cycle as follows.

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-1,$
$f(v_n) = 0.$

The labeling pattern of the vertices $v_{1j}, v_{2j}, ..., v_{nj}$, where $1 \leq j \leq m$ is divided into following subcases.

Subcase 1: $m = 1$.
$f(v_{i1}) = 0; \quad i \equiv 2(mod4);$
$f(v_{i1}) = a; \quad i \equiv 1(mod4);$
$f(v_{i1}) = b; \quad i \equiv 0(mod4);$
$f(v_{i1}) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-1,$
$f(v_{n1}) = a.$

Subcase 2: $m \equiv 0(mod4)$.
$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-1$ and $2 \leq j \leq m,$
$f(v_{nj}) = 0.$

Subcase 3: $m \equiv 1(mod4)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-1$ and $2 \leq j \leq m,$
$f(v_{nj}) = a.$

Subcase 4: $m \equiv 2(mod4)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-1$ and $2 \leq j \leq m,$
$f(v_{nj}) = b.$

Subcase 5: $m \equiv 3(mod4)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-1$ and $2 \leq j \leq m,$
$f(v_{nj}) = c.$

Case 3: $n \equiv 2(mod4)$.

Label the vertices $v_1, v_2, ..., v_n$ of cycle as follows.

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-2,$
$f(v_{n-1}) = 0;$
$f(v_n) = c.$
The labeling pattern of the vertices $v_{1j}, v_{2j}, ..., v_{nj}$, where $1 \leq j \leq m$ is divided into following subcases.

Subcase 1: $m = 1$.

$f(v_{i1}) = 0; \quad i \equiv 2(mod4);$
$f(v_{i1}) = a; \quad i \equiv 1(mod4);$
$f(v_{i1}) = b; \quad i \equiv 0(mod4);$
$f(v_{i1}) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n-1,$
$f(v_{n1}) = b.$

Subcase 2: $m \equiv 0(mod2)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n$ and $2 \leq j \leq m$.

Subcase 3: $m \equiv 1(mod2)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-2$ and $2 \leq j \leq m$,
$f(v_{(n-1)j}) = b;$
$f(v_{nj}) = c.$

Case 4: $n \equiv 3(mod4)$.

Label the vertices $v_1, v_2, ..., v_n$ of cycle as follows.

$f(v_i) = 0; \quad i \equiv 0(mod4);$
$f(v_i) = a; \quad i \equiv 1(mod4);$
$f(v_i) = b; \quad i \equiv 2(mod4);$
$f(v_i) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n.$

The labeling pattern of the vertices $v_{1j}, v_{2j}, ..., v_{nj}$, where $1 \leq j \leq m$ is divided into following subcases.

Subcase 1: $m = 1$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 0(mod4);$
$f(v_{ij}) = c; \quad i \equiv 3(mod4); \quad 1 \leq i \leq n.$

Subcase 2: $m \equiv 0(mod8)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-3$ and $2 \leq j \leq m,$
$f(v_{(n-2)j}) = b;$
$f(v_{(n-1)j}) = c;$
$f(v_{nj}) = a.$

Subcase 3: $m \equiv 1(mod8)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-1$ and $2 \leq j \leq m,$
$f(v_{nj}) = c.$

Subcase 4: $m \equiv 2(mod8)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-3$ and $2 \leq j \leq m,$
$f(v_{(n-2)j}) = 0;$
$f(v_{(n-1)j}) = a;$
$f(v_{nj}) = b.$

Subcase 5: $m \equiv 3(mod8)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-3$ and $2 \leq j \leq m,$
$f(v_{(n-2)j}) = b;$
$f(v_{(n-1)j}) = 0;$
$f(v_{nj}) = c.$

Subcase 6: $m \equiv 4(mod8)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-3$ and $2 \leq j \leq m,$
$f(v_{(n-2)j}) = a;$
$f(v_{(n-1)j}) = c;$
$f(v_{nj}) = 0.$

Subcase 7: $m \equiv 5(mod8)$.

$f(v_{ij}) = 0; \quad i \equiv 2(mod4);$
$f(v_{ij}) = a; \quad i \equiv 1(mod4);$
$f(v_{ij}) = b; \quad i \equiv 3(mod4);$
$f(v_{ij}) = c; \quad i \equiv 0(mod4); \quad 1 \leq i \leq n-3$ and $2 \leq j \leq m,$

$f(v_{(n-2)j}) = b$;
$f(v_{(n-1)j}) = 0$;
$f(v_{nj}) = c$.
Subcase 8: $m \equiv 6(mod8)$.

$f(v_{ij}) = 0$; $\quad i \equiv 2(mod4)$;
$f(v_i) = a$; $\quad i \equiv 1(mod4)$;
$f(v_{ij}) = b$; $\quad i \equiv 3(mod4)$;
$f(v_{ij}) = c$; $\quad i \equiv 0(mod4)$; $\quad 1 \le i \le n-3$ and $2 \le j \le m$,
$f(v_{(n-2)j}) = c$;
$f(v_{(n-1)j}) = a$;
$f(v_{nj}) = b$.

Subcase 9: $m \equiv 7(mod8)$.

$f(v_{ij}) = 0$; $\quad i \equiv 2(mod4)$;
$f(v_{ij}) = a$; $\quad i \equiv 1(mod4)$;
$f(v_{ij}) = b$; $\quad i \equiv 3(mod4)$;
$f(v_{ij}) = c$; $\quad i \equiv 0(mod4)$; $\quad 1 \le i \le n-3$ and $2 \le j \le m$,
$f(v_{(n-2)j}) = 0$;
$f(v_{(n-1)j}) = b$;
$f(v_{nj}) = a$.

The *Table 4* shows that the labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for V_4-cordial labeling. Hence, the Corona $C_n \bigodot mK_1$ is V_4-cordial for all n and m.

Let $n = 4p + q$ and $(j = 1, 2, 3, ..., m)$, where $m, p, q \in N \cup \{0\}$.

Table 4

q	m	Vertex conditions	Edge conditions
0	$m \equiv 0,1(mod2)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(b) = e_f(c)$ $e_f(b) = e_f(c)$
1	$m \equiv 0(mod4)$	$v_f(0) = v_f(a) + 1$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) = e_f(a) + 1$ $e_f(b) + 1 = e_f(c) + 1$
	$m \equiv 1(mod4)$	$v_f(0) = v_f(a)$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) = e_f(a)$ $e_f(b) + 1 = e_f(c) + 1$
	$m \equiv 2(mod4)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c) + 1$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c) + 1$
	$m \equiv 3(mod4)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c)$
2	$m \equiv 0(mod2)$	$v_f(0) = v_f(a)$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) + 1 = e_f(a)$ $e_f(b) + 1 = e_f(c)$
	$m \equiv 1(mod2)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c)$
3	$m \equiv 0(mod8)$	$v_f(0) + 1 = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) + 1 = e_f(a)$ $e_f(b) = e_f(c)$
	$m \equiv 1(mod8)$	$v_f(0) + 1 = v_f(a)$ $v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1$ $e_f(b) = e_f(c) + 1$
	$m \equiv 2(mod8)$	$v_f(0) + 1 = v_f(a)$ $v_f(b) + 1 = v_f(c) + 1$	$e_f(0) + 1 = e_f(a)$ $e_f(b) + 1 = e_f(c) + 1$
	$m \equiv 3,7(mod8)$	$v_f(0) = v_f(a)$ $v_f(b) = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) = e_f(c)$
	$m \equiv 4(mod8)$	$v_f(0) = v_f(a)$ $v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a)$ $e_f(b) + 1 = e_f(c)$
	$m \equiv 5(mod8)$	$v_f(0) = v_f(a) + 1$ $v_f(b) + 1 = v_f(c)$	$e_f(0) = e_f(a) + 1$ $e_f(b) + 1 = e_f(c)$
	$m \equiv 6(mod8)$	$v_f(0) + 1 = v_f(a) + 1$ $v_f(b) + 1 = v_f(c)$	$e_f(0) + 1 = e_f(a) + 1$ $e_f(b) + 1 = e_f(c)$

Illustration 2.8 The corona graph $C_4 \bigodot 5K_1$ and its V_4-cordial labeling is shown in *Fig. 4*.

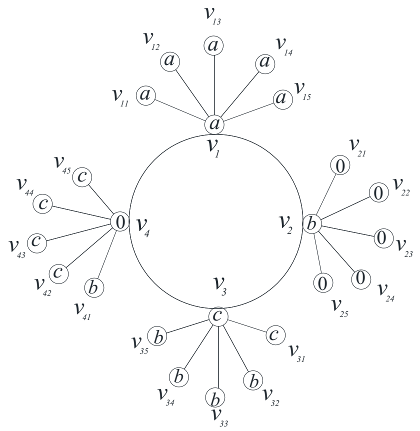

Fig. 4 V_4**-cordial labeling of corona** $C_4 \bigodot 5K_1$

3 Concluding Remarks

From the above study it can be concluded that a Graph can be labeled by either of the two techniques (4-cordial and $V4$-cordial) in some cases may be by both of them and in certain cases may be by neither of them. Thus 4-cordial and $V4$-cordial labeling techniques do not show any inter-dependence and can be considered to be two distinct graph labeling techniques. Following are illustrative examples of the mentioned facts.

1. The Path P_4 is 4-cordial proved by M. Hovey [6] but not V_4-cordial proved by M. Seenivasan and A. Lourdusamy [4].

2. The Star $K_{1,n}$ are 4-cordial proved by M. Hovey [6] and V_4-cordial proved by A. Riskin [3].

3. The Cycle C_4 is neither 4-cordial proved by M. Hovey [6] nor V_4-cordial proved by A. Lourdusamy [4].

Competing interests

Authors have declared that no competing interests exist.

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