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On the Optimization of Transportation Problem

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Short Research Article

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Abstract

The Transportation Problem which deals with the distribution of commodities from a variety of sources to a variety of destinations was considered in this research. In this work, existing theorems such as the duality theorems and complementary slackness theorem were used to analyse the transportation problem and their applicability was observed. Illustration was made with data gathered from a real-life production company (Owerri, Port-Harcourt and Enugu plants). The data collected was modeled as a Linear Programming Problem of the transportation type and solved with TORA optimization software (VAM-MODI Method) to generate an optimal and feasible solution. It was observed that the cost of transportation of finished Returnable Glass Bottle products of the company for a month was in general reduced by 11.58%.

Keywords: Transportation problem; simplex method; linear programming problem; optimization; duality.

1 Introduction

Low productivity and dwindling economy of Nigeria has caused businesses and industries to optimize economically. Transportation Problems are a special case of Network flow problems (a particular class of linear programming) which involves moving commodities from a number of sources to a number of destinations. The economist will always maintain that until the goods and services get to the final consumer, the production process is not yet complete. The objective of a transportation problem is to ensure that demand requirements are fully satisfied at the destination with regards to the available resources at the source/origin, at the least possible cost.

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There are many modes of transportation: air, road, rail, water. Road transport still leads the pack and is a favourite choice owing to its low cost, high flexibility and faster response time. However, it is a danger to world life as it is a major consumer of petroleum products and thus a great contributor to greenhouse gas emissions, CO_2 emission and a lot of other pollutant particles in the air. The transportation method can be used to reduce the impact of using fossil fuels to transport materials. The transportation model can also be extended to other areas of operation, including but not limited to, production-inventory control and equipment maintenance, by recognizing the parallels between the elements of the problem and the transportation model. We also have least-time transportation problems which objective is to minimize time rather than cost, as is usually encountered in hospital management, military and fire services.

Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly, e.g. the Simplex method, but a more efficient solution procedure (algorithm) called the transportation technique, which is hinged on the Simplex method has been developed.

The Transportation Problem has been studied extensively by many authors and has found applications in diverse fields. The first was by Tolstoi [1]. He ordered the allocations based on the difference between the destinations and the sources he used for the transportation of salt, cement, and other cargo between sources and destinations along the railway network of the Soviet Union. [2,3] contributed to the development of the transportation methods involving a number of shipping sources and a number of destinations. Optimal solutions could only be achieved when George B. Dantzig applied the concept of linear programming in solving transportation models [4].

The floating point method for finding an optimal solution to a transportation problem with additional constraints was proposed [5]. The proposed method differed from the existing methods namely, simplex method and inverse matrix method in terms of its ease of use and efficiency. But the optimum solution obtained is not necessarily feasible i.e. targeted at having $(m+n-1)$ allotted entries. Also proposed was the dual matrix approach which is very efficient in terms of computation than the simplex method as it can be applied to both the balanced and unbalanced TP [6]. This approach considers the dual of the transportation model instead of its primal and removes the problem of degeneracy in solution as it does not require path-tracing. The approach however needs an $(m+n) \times (m+n)$ matrix which is quite large and disadvantageous.

In comparing the direct methods for finding optimal solution of a transportation problem, it was summarized that the VAM-MODI method is the best method to rely on as it consistently gave correct results [7], hence our choice of the VAM-MODI for our own work. Other researchers in this field include [8,9,10,11,12].

Based on these reviews which we have studied and noted some of their short-comings, we will carry out our own study by using the Two-phase method and VAM-MODI Transportation Technique and application of the knowledge of some basic theorems to solve a real life economic problem.

2 Mathematical Formulation, Methodology and Data Presentation

The VAM-MODI transportation method is a modification of the Simplex method which is enshrined in LP-Duality. At optimality, the values of the objective functions for a primal problem and its corresponding dual problem are equal. Generally speaking, the two problems are not solved with the same degree of ease; and therefore there is a preference, quite marked in some cases, as to which form of the problem will be used for the solution. Solving a dual instead of the primal could have computational advantages if the number of variables in the primal is considerably smaller than the number of constraints because the amount of simplex computations depends largely on the number of constraints. Thus we solve the dual and determine the primal from the result obtained. We take a look at a few duality theorems.

Theorem 1 (Weak duality theory) [13]

Given a primal LPP

$$\left.\begin{array}{l} Max.\ Z = C^T x \\ Subject\ to:\ Ax \leq b \\ \quad x \geq 0 \end{array}\right\} \tag{2.1}$$

And its' dual

$$\left.\begin{array}{l} Min.\ Z' = b^T y \\ Subject\ to:\ A^T y \geq C \\ y \geq 0 \end{array}\right\} \tag{2.2}$$

Then for any feasible solutions x_0 and y_0 to the primal and dual linear programs respectively, then $C^T x_0 \leq b^T y_0$.

The relationship does not specify which is primal and which is dual. Only the sense of optimization (maximization or minimization) is important in this case. (Objective value in the maximization problem) ≤ (Objective value in the minimization problem) for any pair of feasible, primal and dual solutions. The weak duality theory provides a bound on the optimal value of the objective function of either the primal or the dual.

Theorem 2 (Strong duality theorem) [14]

Consider a primal LPP (2.1) and its dual (2.2). Let x_0 and y_0 be feasible solutions to the primal and dual linear programs respectively. If $C^T x_0 = b^T y_0$, then both x_0 and y_0 are optimal solutions to their respective problems.

Remarks:

The optimal value of the primal and the dual problems are in fact equal i.e. if either (2.1) or (2.2) has a finite optimal value, then so does the other; the optimal values coincide and optimal solutions to both exist.

Theorem 3 (Complementary slackness) [14]

Let x_0 be a feasible solution to (2.1) and y_0 be a feasible solution to (2.2). Then x_0 is optimal to the primal (2.1) and y_0 is optimal to the dual (2.2) if and only if the conditions of complementary slackness hold. i.e.

$$\left(b_i - \sum_{j=1}^{n} a_{ij} x_j\right) y_i = 0 \qquad for \quad 1 \leq i \leq m$$

and

$$\left(\sum_{i=1}^{m} a_{ji} y_i - C_j\right) x_j = 0 \qquad for \quad 1 \leq j \leq n$$

Remarks:

Complementary slackness relations are used to check whether a proposed solution is optimal or not, and also to give a uniqueness result. If in an optimal solution of a LPP the value of the dual variable (shadow price) associated with a constraint is non-zero, then that constraint must be satisfied with equality. Furthermore, if a constraint is satisfied with strict inequality, then its corresponding dual variable must be zero. In $Ax = b$, if the rows of A are linearly independent, then $Ax = b$ has a unique solution $x \in \mathbb{R}^N$. The finiteness of the optimal value implies the existence of a solution. Complementary conditions guarantee that the values of the primal and dual are the same.

Consider a typical transportation problem

$$\left.\begin{aligned} Min. Z = & \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ subject\ to: & \quad \sum_{j=1}^{n} x_{ij} \le a_i \quad , \qquad i = 1,2,...,m \\ & \quad \sum_{i=1}^{m} x_{ij} \ge b_j \quad , \qquad j = 1,2,\ldots,n \\ & \quad x_{ij} \ge 0 \qquad \forall \quad i,j \end{aligned}\right\} \tag{2.3}$$

We want to establish the properties of the objective function $Z(x)$.

We define $\quad Z(x): X \subseteq \mathbb{R} \rightarrow \mathbb{R}$

Now $\quad X = \{x_{ij}\} \, \forall \, i,j$

$\quad X = \{x_{11}, x_{12}, \ldots, x_{1n}, x_{21} \ldots \, x_{mn}\}$

Let $P = \{p\!: p\ is\ the\ production\ capacity\ of\ the\ various\ plants\}$

There is a limit to the number of goods produced at each factory i.e. each plant has its production capacity. Also, of all the plants, there is one with the highest production capacity and another with the lowest production capacity. These capacities therefore constitute the highest and lowest values for this set. Then we can say that P is bounded.

But $X \subset P$. Thus X is bounded (since it is contained in a bounded interval).

Consider also $X = \{x_{11}, x_{12}, \ldots, x_{1n}, x_{21} \ldots \, x_{mn}\}$ which we can say are points. Picking a particular point or value, say, x_{ij} , then its complement will be

$$\{x_{ij}\}^c = \{x_{ij} \in X : -\infty < x_{ij}\} \cup \{x_{ij} \in X : x_{ij} < \infty\}$$

i.e. $\quad \{x_{ij}\}^c = \mathbb{R} - \{x_{ij}\} = (-\infty, x_{ij}) \cup (x_{ij}, +\infty)$

Finite union of open sets is open, therefore $\{x_{ij}\}^c$ is open. Hence $\{x_{ij}\}$ is closed. The union of finite collection of closed sets is also closed. Therefore, X is closed.

Having ascertained that X is bounded and also closed, we therefore conclude that X is compact. Hence, the domain of our objective function is compact.

Consider again our objective function $Z(x) = \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ in equation (2.3)

$$Z(x): X \subseteq \mathbb{R} \rightarrow \mathbb{R}$$

Let $x, y \in X$ and α, β scalars in $\mathbb{R}$.
To show linearity,

$$Z(\alpha x + \beta y) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}(\alpha x + \beta y)_{ij}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}(\alpha x_{ij} + \beta y_{ij})$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} (\alpha C_{ij} x_{ij} + \beta C_{ij}\, y_{ij})$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha C_{ij} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \beta C_{ij} y_{ij}$$

$$= \alpha \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} x_{ij} + \beta \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} y_{ij}$$

$$= \alpha Z(x) + \beta Z(y)$$

Hence, our objective function $Z(x)$ is a linear function.

Theorem 4: Every linear function in a finite dimensional space is continuous [15].

Remark: By theorem 4, $Z(x)$ is then continuous.

We have already established that our function $Z(x)$ is linear and continuous on the closed and bounded interval.

Theorem 5 (Maximum and Minimum Value Theorem) [15]

Every continuous function defined on a compact space say Y attains its minimum and maximum on the same space Y. i.e. if f is continuous, then there exists m and M such that $m \leq f(x) \leq M \quad \forall\, x \in Y$.

Hence, by theorem 5, our objective function $Z(x)$ attains its minimum and maximum values on the same interval. It is based on these verified observations that we can begin to talk about optimizing any given function. This thus provides a rationale for carrying out our optimization work.

2.1 Solution algorithm for the transportation problem

Transportation models do not start at the origin where all decision values are zero; they must instead be given an initial feasible solution. The solution algorithm to a transportation problem can be summarized in the following steps:

1. Formulate the problem, make it a standard problem and put it in the tableau form.
2. Obtain an initial basic feasible solution using any of the following methods: North West Corner Rule, Minimum (Least) Cost Method, Vogel's Approximation Method.
3. Test the initial solution for optimality using either the Stepping Stone Method or the Modified Distribution Method (MODI). If the solution is optimal then stop, otherwise, determine a new improved solution by repeating step 3 until an optimal solution is reached.
4. Calculate the optimal transportation cost and determine the basic variables.

The transportation technique uses the steps of the simplex method but differs only in the implementation of the optimality and feasibility conditions. The VAM-MODI method is used in this work and the steps are as listed below:

2.2 Vogel's approximation method (VAM)

Presented by Dr. William R. Vogel, it is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (or penalty) costs, hence it is sometimes referred to as the penalty method or the regret method. The steps involved in determining an initial solution using VAM are as follows;

1. Write the given transportation problem in tabular form.
2. Determine the difference between the minimum cost and the next minimum cost corresponding to each row and each column. This is known as penalty cost.
3. Identify the row or column having the highest penalty cost, breaking ties arbitrarily. Then select the cell with lowest cost in the identified row or column and assign as many units of the product as possible to it.
   Reduce the row supply and column demand by the number of units assigned to the cell and cross out the row supply or column demand that is satisfied; then form a new tableau. If a row and a column are satisfied simultaneously, only one of them may be crossed out and the remaining row (or column) is assigned a zero supply (or demand). Any row or column with zero supply or demand should not be used in calculating penalties further.
4. Re-compute the row and column penalties for the reduced transportation tableau as in step 2 above. Then go to step 3. Continue in this manner until all the supply and demand requirements are met.
5. If exactly one row or column remains uncrossed out, stop. If only one row (or column) with positive supply (or demand) remains uncrossed out, determine the basic variables in the row (or column) by the least-cost method. If all the uncrossed-out rows and columns have zero supply and demand, determine the zero basic variables by the least-cost method.

An optimal solution of a transportation problem is one where there is no other set of transportation routes that will further reduce the total transportation cost. To obtain an optimal solution, we make successive improvements to the initial basic feasible solution until no further decrease in the transportation cost is possible.

2.3 The modified distribution method (MODI)

MODI is a modified version of the stepping-stone method in which mathematical equations are used in place of the stepping-stone paths. Sometimes referred to as the *u-v* method, it is a substantial improvement over the stepping stone method, and thus mostly used for computation. The steps involved in this method are as follows;

1. Obtain an initial basic feasible solution using any of the three methods mentioned before.
2. Improve the initial basic feasible solution obtained in step 1 above by first calculating the shadow (aka marginal) prices u_i and v_j (i=1,2, . . . , m; j=1,2, . . ., n) for the demand and supply centres of the basic variables obtained in step 1 above, noting that $u_i + v_j = c_{ij}$ for each basic x_{ij} and setting $u_1 = 0$. This can be done directly on the transportation tableau, or by writing the (u,v)-equations explicitly.
   u_i and v_j represent the value of the commodity at each cell. The value of each u_i measures the comparative implicit contribution of a unit of product at the supply centre i and may be called the location rent, while the value of each v_j measures the comparative implicit contribution of an additional unit of the commodity shipped to demand centre j and hence may be termed the market price (i.e. delivered market value of the commodity at demand destination).

3. Calculate the potential benefits l_{ij} of each of the non-basic variables. Again note that

$l_{ij} = c_{ij} - u_i - v_j$ for every unoccupied cell. The non-basic variable with the most negative potential benefit value becomes the entering variable. Break ties arbitrarily.

4. Construct a loop for the entering variable. The loop starts with cell of the entering variable where it has a positive sign and must end in the same cell of the entering variable. All corners of the loop lie in the cells for which allocations have been made, and the loop may skip over any number of occupied or vacant cells. All occupied cells at the corners of the loop take alternate signs.
5. The basic variable with the least allocation value among the corners having a negative sign in the loop becomes the leaving variable. We thus increase the corners with a positive sign by the allocations previously assigned to the leaving variable and reduce the corners with a negative sign by the same amount, always maintaining the row and column requirements i.e. that supply limits and demand requirements remain satisfied and that all shipments through all routes must be nonnegative.
6. This gives us an improved feasible solution. Repeat steps 2 to 5 above till there is no more negative potential benefit. The test for optimality thus terminates and the present feasible solution is optimal.
7. If in an optimal solution any l_{ij} = 0, then the current basic feasible solution is not unique though optimal. As such, alternate solution with the same optimal value exists.

2.4 Data presentation and analysis

For effective research, this work will be limited to the Company plants in the Eastern Region namely Owerri, Enugu and Port-Harcourt, and the depots that these plants supply goods to. This work concentrates on the development of an optimization model for the shipping of the Returnable Glass Bottle (RGB) products of this firm, in the Eastern region of Nigeria (3 plants and 19 depots), in the best cost-effective manner. This study is intended to fit a model for the distribution data of Returnable Glass Bottle (RGB) products of the Company (Owerri, Enugu and Port-Harcourt) as a linear transportation problem and to minimize the transportation cost, hence determining a minimum cost schedule for distributing these products from the three sources to the different destinations. The cost saved will be channeled to the actualization of the company's other projects especially with regards to its Corporate Social Responsibility.

We concentrated on the transportation of a single product, Returnable Glass Bottle (RGB). The data for this study was gathered for the period Jan 2012 – Dec 2012. One month planning horizon was adopted for convenience as wages and salaries are mostly paid on monthly basis in Nigeria. The following assumptions were made in the model formulation:

1. The cost of transporting a unit of the product c_{ij} is independent of the number of units of the product x_{ij} shipped. (i.e no volume discount is given)
2. For every round trip, the haulage truck carries its maximum load.
3. Only direct shipments are allowed between a source and a destination
4. The haulage trucks have the same load capacity, technical condition, etc.
5. A linear model is assumed
6. Cost of production is uniform at all plants

Data collected from the three plants are combined to formulate a balanced transportation tableau as shown in Table 1.

3 Results and Discussion

The data obtained in Table 1 was solved using the TORA optimization software to obtain the final iteration as shown in Table 2. The final iteration gave an objective value of ₦ 42,196,521.52k.

The solution shows that the minimum monthly total transportation cost is ₦ 42,196,521.52k. The values for the decision variables show the quantity of the product RGB to ship over each route. This optimal solution saves this company a transportation cost of ₦ 217,543.10 (Two hundred and seventeen thousand, five hundred and forty-three naira ten kobo) every month. Note that $m + n - 1 = 3 + 20 - 1 = 22$ used routes. Hence this solution is also not degenerate.

1. We have a positive slack variable (94809 units) associated with constraint 1. Therefore the constraint is non-binding and does not restrict the possible changes of the point. We have not used all the resource available to us, and so small changes in the right hand side do not affect the optimal solution (i.e. the solution is not dependent on the constraint). This can also be seen as the shadow price associated with the constraint 1 is zero and that means there is an infinite allowable increase. A unit increase or decrease in this constraint will not affect the objective value. All other constraints are binding, with the resultant shadow prices attached to them.
2. Shadow price (or dual price): From the values obtained, we see that a one unit increase in the constraint 4 (i.e. demand at Orlu depot) increases the objective value by N14.45. In general, the dual prices of constraints 4-22 are all positive. Thus increasing the right hand side will lead to increased objective value which is not our aim in this work. The shadow prices of constraints 2 and 3 are negative, indicating that the objective value can be reduced for every proportionate increase in the right hand side of the constraints, and this is the bases for our analysis I and II below. Also, the dual price of constraint 1 is zero which reveals that there is no economic advantage in increasing the supply / production capacity of Owerri plant. However, we advice a reduction in production to save available resources and manpower thereby reducing production cost.
3. Reduced cost or opportunity cost: It is observed from our result, that the reduced cost for the non-basic variables are all positive, indicating that they are not eligible to enter the basis (i.e. not wanted) as an increase in the variables will result in an increase in its objective function coefficient, and since we are solving a minimization problem, we are not going to increase the quantity of the variables so that we do not incur more transportation cost. Again, we do not have an alternate optimal solution here as there is no basic variable with an optimal value of zero and a corresponding reduced cost value of zero also.

3.1 Sensitivity analysis

Conducting businesses is a very dynamic process. Economic forces constantly change the environment in which a firm procures inputs and markets its outputs, and so it is very necessary that we determine how the solution obtained changes when some or all parameters of the model developed are varied. These changes may pertain to costs, quantity produced by (or distributed to) one or more factories (or locations). After a problem has been completely solved, it is often advantageous to investigate the effect of a change in some or all of the parameters of the solution.

3.1.1 Analysis I

In this analysis, the supplies from Owerri and Enugu plants are reduced to 522,676 and 570,763 cases respectively, while that of Port-Harcourt is increased to 701,548 cases. The LPP was re-solved and the following schedule as shown in Table 3 was obtained;

The computer solution of this first sensitivity analysis carried out shows a minimum total transportation cost of ₦ 39,576,238.33. This yields a 6.21% reduction in monthly transportation cost for the Company.

3.1.2 Analysis II

The production of RGB was further reduced to 315576 cases in the Owerri plant, while that of Enugu and Port-Harcourt were increased to 670282 and 809129 cases respectively. After computations, an optimal solution with a total minimum transportation cost of ₦38,544,841.80k, which is an 8.65% reduction in monthly transportation cost was obtained. Below in Table 4 is the obtained schedule.

Table 1. Balanced transportation tableau for the three plants

	Orlu 1	Umuahia 2	Ikot Ekpene 3	Uyo 4	Eket 5	Calabar 6	ABA 7	Urban Enugu 8	Awka 9	Nsukka 10	Abakaliki 11	Makurdi 12	Wukari 13	Otukpo 14	Gboko 15	Nnewi 16	Ahoada 17	Eleme 18	Elechi 19	Dummy 20	Supply
Owerri 1	14.45	16.67	34.03	37.47	44.60	51.75	20.57	31.16	27.44	44.32	51.30	63.91	96.37	63.18	69.02	21.08	30.26	34.09	21.53	0.00	725,349
Enugu 2	31.16	29.92	37.02	40.29	36.94	47.17	34.32	10.70	14.40	21.62	21.00	43.76	87.18	29.86	53.60	20.69	33.99	40.09	47.06	0.00	613,638
Port Harcourt 3	21.53	20.69	26.71	27.05	23.85	34.32	21.45	47.06	43.47	41.19	36.97	78.41	70.19	47.62	56.96	27.78	16.84	14.31	11.10	0.00	456,000
Demand	70,638	54,730	65,278	110,523	97,478	105,195	95,399	186,389	82,971	90,569	64,966	61,115	42,875	23,144	67,455	93,673	155,040	108,000	124,740	94,809	

(Table showing the demand, supply and unit cost data for the three plants)

Table 2. Final iteration

	Orlu 1	Umuahia 2	Ikot Ekpene 3	Uyo 4	Eket 5	Calabar 6	ABA 7	Urban Enugu 8	Awka 9	Nsukka 10	Abakaliki 11	Makurdi 12	Wukari 13	Otukpo 14	Gboko 15	Nnewi 16	Ahoada 17	Eleme 18	Elechi 19	Dummy 20	Supply
Owerri 1	70,638	54,730	65,278	110,523			95,399									93,673	15,559		124,740	94,809	725,349
Enugu 2						37,029		186,389	82,971	90,569	64,966	61,115		23,144	67,455						613,638
Port Harcourt 3					97,478	68,166							42,875				139,481	108,000			456,000
Demand	70,638	54,730	65,278	110,523	97,478	105,195	95,399	186,389	82,971	90,569	64,966	61,115	42,875	23,144	67,455	93,673	155,040	108,000	124,740	94,809	

Table 3. Table showing the final iteration for Analysis I

	Orlu 1	Umuahia 2	Ikot Ekpene 3	Uyo 4	Eket 5	Calabar 6	ABA 7	Urban Enugu 8	Awka 9	Nsukka 10	Abakaliki 11	Makurdi 12	Wukari 13	Otukpo 14	Gboko 15	Nnewi 16	Ahoada 17	Eleme 18	Elechi 19	Dummy 20	Supply
Owerri 1	70,638	54,730	65,278	42,303			95,399		5,846							93,673				94,809	725,349
Enugu 2								186,389	77,125	90,569	64,966	61,115		23,144	67,455						613,638
Port Harcourt 3				68,220	97,478	105,195							42,875				155,040	108,000	124,740		456,000
Demand	70,638	54,730	65,278	110,523	97,478	105,195	95,399	186,389	82,971	90,569	64,966	61,115	42,875	23,144	67,455	93,673	155,040	108,000	124,740	94,809	

Table 4. Table showing the final iteration for Analysis II

	Orlu 1	Umuahia 2	Ikot Ekpene 3	Uyo 4	Eket 5	Calabar 6	Aba 7	Urban Enugu 8	Awka 9	Nsukka 10	Abakaliki 11	Makurdi 12	Wukari 13	Otukpo 14	Gboko 15	Nnewi 16	Ahoada 17	Eleme 18	Elechi 19	Dummy 20	Supply
Owerri 1	70,638	54,730	0				95,399									0				94,809	725,349
Enugu 2								186,389	82,971	90,569	64,966	61,115		23,144	67,455	93,673					613,638
Port Harcourt 3			65,278	110,523	97,478	105,195							42,875				155,040	108,000	124,740		456,000
Demand	70,638	54,730	65,278	110,523	97,478	105,195	95,399	186,389	82,971	90,569	64,966	61,115	42,875	23,144	67,455	93,673	155,040	108,000	124,740	94,809	

Note that routes Owerri - Ikot Ekpene and Owerri – Nnewi have been given zero allocation; this means that though we have indicated these routes as being in use, no shipment is made on these routes. Thus some basic variables in our current solution have zero values and hence, the solution even though it is optimal, is therefore degenerate.

Full production capacity has already been achieved at the Port-Harcourt plant. But in order to achieve this 8.65% reduction in transportation cost, the port-Harcourt plant needs to work overtime or increase their production capacity by getting a bigger plant. Production should be increased at Enugu plant to meet the requirements. The Owerri plant should also reduce its production of RGB accordingly, and channel its resources to the production of other products, say, 'Table water' which these other plants do not produce or send some of their personnel to work in their Port-Harcourt plant.

4 Summary, Conclusion and Recommendation

The transportation cost is an important element of the total cost structure for any business. Transportation sector is the irreplaceable infrastructure upon which economic and social development is possible. Based on the results and findings of this study, I recommend to the management of this Company especially in Owerri, Enugu and Port-Harcourt to apply mathematical theories in their operations as it is a necessary tool for decision making, not only in logistics (the transportation Problem), but other areas of production as well as administration. If the proposed transportation model is employed, it will greatly assist in devising an efficient distribution plan thereby minimizing transportation cost.

In this study, we have been able to solve a real-life economic problem by developing a model for the transportation schedule in Owerri, Port-Harcourt and Enugu plants of a Company. We have been able to establish the existence of a solution to the model formulated. We have considered three plants and the depots they supply in the Eastern region of Nigeria. We have been able to obtain an optimal transportation schedule for them, thus reducing the amount of money spent by the company on the distribution of finished products by 0.5% (₦ 217,543.10) per month. Also considering the analyses carried out, the transportation cost can be reduced to 6.21% and further by 8.65% even though this last solution is degenerate.

Presently, the Owerri Plant supplies only 7 depots, Enugu plant supplies only 9 depots while Port-Harcourt supplies only 3 depots at a total monthly transportation cost of ₦44,758,990.02k. In this study, we were able to obtain an optimal transportation schedule for our case study, reducing the amount of money spent by the company on the distribution of finished products in general by 11.58% per month. The model developed in this study is adaptable to other companies and businesses.

Competing Interests

Authors have declared that no competing interests exist.

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