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The Very Cost Effective Graph Folding of the Join of Two Graphs

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Authors' contributions

This work was carried out in collaboration between both authors. Author EMEK designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author HA managed the analyses of the study and managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper we studied the very cost effective graph property for the join graph of two graphs. In general this is may or may not be a very cost effective graph. We obtained the conditions for the join graph of two graphs to be a very cost effective graph. First we proved that the join graph $P_n \vee P_m$ of path graphs is very cost effective graph if $n+m$ is an even number and is not if $n+m$ is an odd number. Then we proved that the join graph of any two cycle graphs C_n and C_m where n, m are both odd is very cost effective, and the join graph $P_n \vee C_n$ is a very cost effective graph if n is an odd number. Also we proved that the join graph $G_1 \vee G_2$ of two very cost effective graphs G_1 and G_2 is a very cost effective graph if $n(G_1)+n(G_2)$ is even. Finally we proved that the graph folding of the join graph of two very cost effective graphs not always very cost effective but this will be the case if the sum of the numbers of the vertices in the image of the graph folding is even.

Keywords: The join graph; graph folding; very cost effective graph.

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1 Introduction

A graph G = (V,E) is a nonempty, finite set of elements called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set of G is denoted by V (G) and the edge set of G is denoted by E(G). A simple graph is a graph with no loops and no multiple edges. If e = $\{u, v\}$ is an edge of a graph G, then u and v are adjacent vertices, while u and e are incident. Two adjacent vertices are called neighbors of each other. The degree (valency) of a vertex v in a graph G is the number of edges incident to v. A vertex of degree 0 in G is called an isolated vertex. A vertex v is said to be even or odd, according to whether its degree in G is even or odd. If all the vertices of a graph G have the same valency then it is called a regular graph. The order of G, denoted n(G) =|V(G)|, is the number of vertices in G. The size of G, denoted m(G) =|E(G)|, is the number of edges in G. A graph of order 1 is called a trivial graph, and a graph of order at least 2 is called a non trivial graph. A graph of size 0 is called an empty graph. A nonempty graph has one or more edges. A graph is said to be connected if every pair of vertices has a path connecting them, otherwise is called disconnected. For any vertex $v \in$ V (G), the open neighborhood of v is the set N(v) = $\{u \in$ V (G) | $uv \in$ E(G)$\}$, and the closed neighborhood of v is the set N[v] =$\{$ N(v)∪ $\{v\}\}$. For a set S ⊆ V (G), its open neighborhood N(S) = $\cup_{v\epsilon S}$ N(v), and its closed neighborhood is N[S] =N(S)∪S. A path P_n is graph in which any two vertices are connected by exactly one edge with two vertices of degree 1, and the other n-2 vertices of degree 2.

Definition (1.1)

Let G_1 and G_2 be simple graphs and f : $G_1 \rightarrow G_2$ be a continuous function . Then f is called a graph map, if

(i) For each vertex v ϵ V(G_1) , f(v) is a vertex in V(G_2).
(ii) For each edge e ϵ E(G_1) , dim(f(e)) ≤ dim(e), [1].

Definition (1.2)

A graph map f :$G_1 \rightarrow G_2$ is called a graph folding iff f maps vertices to vertices and edges to edges ,i.e., for each vertex v ϵV (G_1) , f(v) is a vertex in V(G_2) and for each edge e ϵ E(G_1) , f(e)is an edge in E(G_2), [2].

Cost effective and very cost effective sets in graphs were introduced in [3] and studied further in [4]. Very cost effective bipartitions were also first introduced in [3] and were motivated by the studies of unfriendly partitions [5].

Definition (1.3)

A vertex v in a set S is said to be cost effective if it is adjacent to at least as many vertices in V \ S as in S, that is,|N(v) ∩ S| ≤ |N(v) ∩ (V \ S)|. A vertex v is very cost effective if it is adjacent to more vertices in V\S than in S, that is, |N(v) ∩ S| < |N(v) ∩ (V \ S)|. A set S is (very) cost effective if every vertex $v \in$ S is (very) cost effective, [3].

Definition (1.4)

A bipartition π = {S, V \ S} is called cost effective if each of S and V \ S is cost effective, and π is very cost effective if each of S and V \ S is very cost effective. Graphs that have a (very) cost effective bipartition are called (very) cost effective graphs, [4].

Note that not every graph has a very cost effective bipartition, e.g., the cycle and the complete graphs of odd orders are not very cost effective.

Definition (1.5)

If G_1 and G_2 are vertex-disjoint graphs. Then the join, $G_1 \vee G_2$, of G_1 and G_2 is a super graph of G_1+G_2, in which each vertex of G_1 is adjacent to every vertex of G_2. The vertex set $V(G_1 \vee G_2)=V_1 \cup V_2$, [6].

2 The Join Graph of Very Cost Effective Graph

We will study the very cost effective graph property for the join graph of two graphs. First we pay attention to path graphs where any path graph is a very cost effective graph [3]. It should be noted that the join graph $P_n \vee P_m$ is not necessarily a very cost effective graph. For example $P_2 \vee P_3$ and $P_4 \vee P_5$ are not very cost effective graphs, see Fig. 2.1.

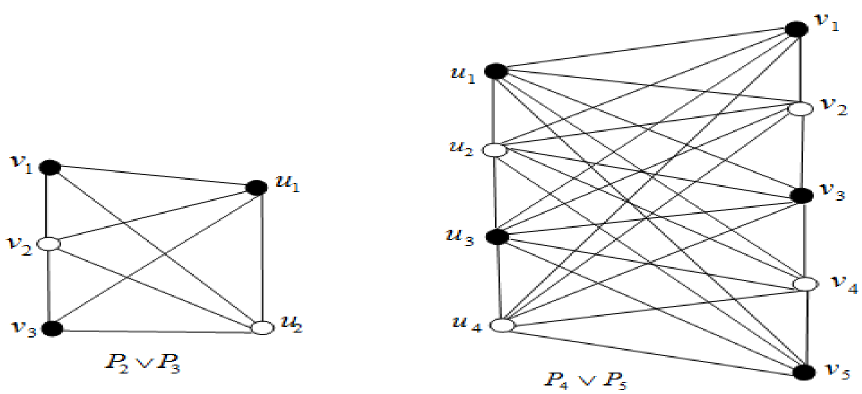


Fig. 2.1

In general this is true if n+m is an odd number, as it is proved in theorem (2.3). The following two lemmas give very cost bipartitions for path graphs P_n , where n is odd or even.

Lemma (2.1)

Any path P_i , $i = 1,2, \dots, n$ where n is even has the only very cost effective bipartition $\pi = \{S , V \setminus S\}$, where $S = \{v_1, v_3, \dots, v_{n-1}\}$ and $V \setminus S = \{v_2, v_4, \dots, v_n\}$. In this case $n(S) = n(V \setminus S)$.

Proof:

Let u be a vertex in S , then $|N(u) \cap S)| = \emptyset$, $|N(u) \cap (V \setminus S)| = 1$ or 2 (1 for an end vertex and 2 otherwise) and hence u is very cost vertex in S . Also, we can prove that $V \setminus S$ is a very cost effective set. Hence π is a very cost effective bipartition, see Fig. 2.2a.

Lemma (2.2)

Any path P_i , $i = 1,2, \dots, n$ where n is odd has the only very cost effective bipartitions $\pi_1 = \{S_1, V_1 \setminus S_1\}$, where $S_1 = \{v_1, v_3, \dots, v_n$ and $V_1 \setminus S_1 = \{v_2, v_4, \dots, v_{n-1}\}$ or $\pi_2 = \{S_2, V_2 \setminus S_2\}$, where $S_2 = \{v_2, v_4, \dots, v_{n-1}\}$, $V_2 \setminus S_2 = \{v_1, v_3, \dots, v_n\}$. In this case either $n(S_1) = n(V_1 \setminus S_1) + 1$ or $n(V_2 \setminus S_2) = n(S_2) + 1$.

Proof:

By the same procedure as lemma (2.1), we can prove this lemma, see Fig. 2.2b.

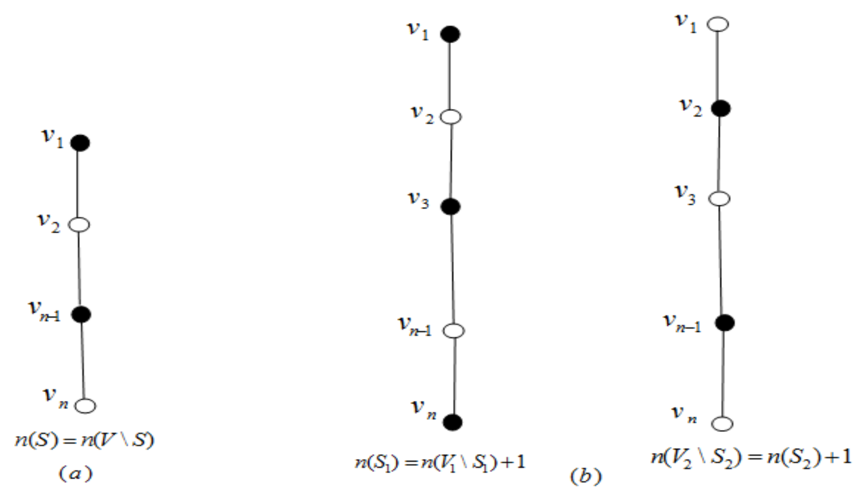


Fig. 2.2

Theorem (2.3)

The join of any path graphs P_n and P_m where $n + m$ is an odd number is not a very cost effective graph.

Proof:

Consider two path graphs P_n and P_m with very cost effective bipartitions $\pi_1 = \{S_1, V_1 \setminus S_1\}$ and $\pi_2 = \{S_2, V_2 \setminus S_2\}$, where $V_1 = V(P_n)$ and $V_2 = V(P_m)$. Without loss of generally, we may assume n is even and m is odd , then $n(S_1) = n(V_1 \setminus S_1)$ and let $n(S_2) = n(V_2 \setminus S_2) + 1$. Let u be an end vertex in S_1 , then u is adjacent to more vertices of $V_1 \setminus S_1$ than in S_1 by one . Now consider the join graph $P_n \vee P_m$ with the bipartition $\pi = \{S, V \setminus S\}$ where $S = S_1 \cup S_2$ and $V \setminus S = (V_1 \setminus S_1) \cup (V_2 \setminus S_2)$, any vertex u in this case will be adjacent to all the vertices of P_m such that $n(N(u) \cap S) = n(S_2)$ and $n(N(u) \cap V \setminus S) = n(V_2 \setminus S_2)$. But $n(S_2) = n(V_2 \setminus S_2) + 1$. This means that any vertex u is adjacent to the same number of any vertices of S_2 and $V_2 \setminus S_2$, and thus the join graph $P_n \vee P_m$ is not a very cost effective graph.

Theorem (2.4)

The join of any path graphs P_n and P_m where $n + m$ an even number is a very cost effective graph.

Proof:

Consider two path graphs P_n and P_m with very cost effective bipartitions $\pi_1 = \{S_1, V_1 \setminus S_1\}$ and $\pi_2 = \{S_2, V_2 \setminus S_2\}$, respectively. We have two cases.

Case 1: n and m are both even

In this case $n(S_1) = n(V_1 \setminus S_1)$ and $n(S_2) = n(V_2 \setminus S_2)$. For any vertex $u \in P_n$, u is adjacent to more vertices in $V_1 \setminus S_1$ than in S_1 .But u in the join graph $P_n \vee P_m$ will be adjacent to the same numbers of vertices of S_2 and $V_2 \setminus S_2$, then u is a very cost effective vertex . This also the case for any vertex $v \in P_m$. Then the join graph $P_n \vee P_m$ is a very cost effective graph.

Case 2: n and m are both odd

Consider the very cost effective bipartitions π_1 and π_2 where $n(S_1) = n(V_1 \setminus S_1) + 1$ and $(V_2 \setminus S_2) = n(S_2) + 1$. Let $u \in S_1$ so u is adjacent to more vertices in $V_1 \setminus S_1$ than in $S_1$by at least one (1 or 2). But in the join graph $P_n \vee P_m$, where $S = S_1 \cup S_2$ and $V \setminus S = (V_1 \setminus S_1) \cup (V_2 \setminus S_2)$, any vertex u will be adjacent to all the vertices of $V_2 \setminus S_2$ and S_2 ,so u is adjacent to more vertices of $V \setminus S$ than in S. Thus u is very cost effective vertex. Now, let $v \epsilon (V_1 \setminus S_1)$, so v is adjacent to more vertices in S_1 than in $V_1 \setminus S_1$ by two. But in the join graph $P_n \vee P_m$ the vertex v will be adjacent to more vertices of S than in $V \setminus S$, so v is a very cost effective vertex , and consequently the join graph $P_n \vee P_m$ is a very cost effective graph.

Example (2.5)

The join graphs $P_2 \vee P_4$ and $P_3 \vee P_5$ are very cost effective graphs, see Fig. 2.3.

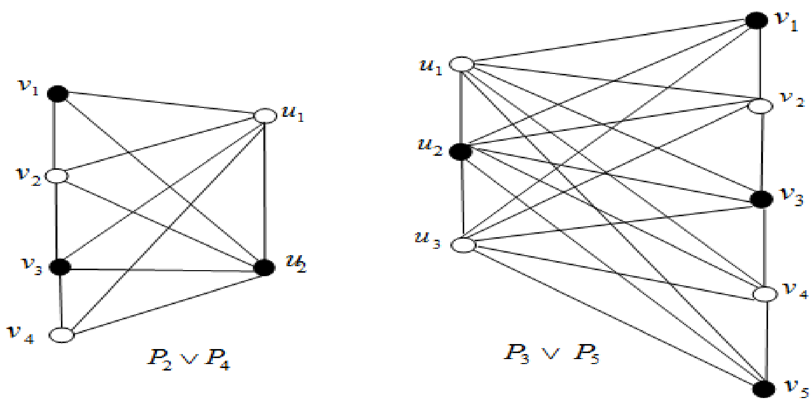


Fig. 2.3

In the following we would like to answer the following questions

1- Is every join graph G_1 v G_2 a very cost effective graph?
2- If G_1 is very cost effective graph, is G_1 v G_2 very cost effective for all graphs?
3- If G_1 and G_2 are both very cost effective graphs, is G_1 v G_2 very cost effective graph?

The answer of the first question is, in general, no, i.e., the join G_1 v G_2 of non trivial connected graphs G_1 and G_2 may or may not a very cost effective graph . For example, the join graph G_1 v C_3 shown in Fig. 2.4 is not a very cost effective graph while the join graph C_3 v C_3 shown in Fig. 2.5 is a very cost effective graph.

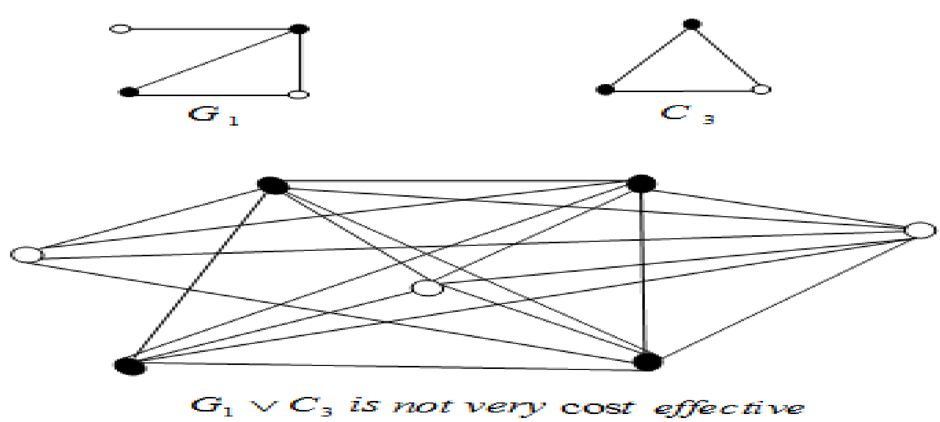
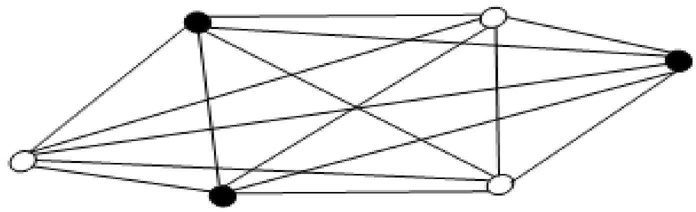


Fig. 2.4



$C_3 \vee C_3$ *is a very cost effective*

Fig. 2.5

The answer of the first question is yes for the following special case of graphs. Before discussing this case it should be noted that no cycle graph C_{2n+1} of odd order is very cost effective [3].

Lemma (2.6)

For a cycle graph C_n , where n is odd the bipartition $\pi = \{S, V \setminus S\}$, where $S = \{v_1, v_2, v_4, \dots, v_{n-1}\}$ and $V \setminus S = \{v_3, v_5, \dots, v_n\}$ is cost effective. In this case $(S) = n(V \setminus S) + 1$.

It is easy to prove any vertex $v_1 \in S$ is cost effective while the other vertices in both S and $V \setminus S$ are very cost effective vertices and hence π is a cost effective bipartition, see Fig. 2.6.

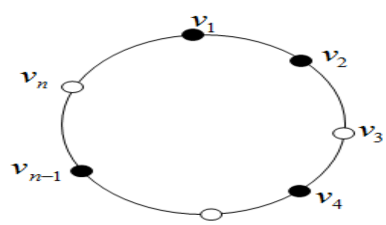


Fig. 2.6

Note that the bipartition $\pi = \{V \setminus S\ ,\ S\}$ is also a cost effective bipartition.

Theorem (2.7)

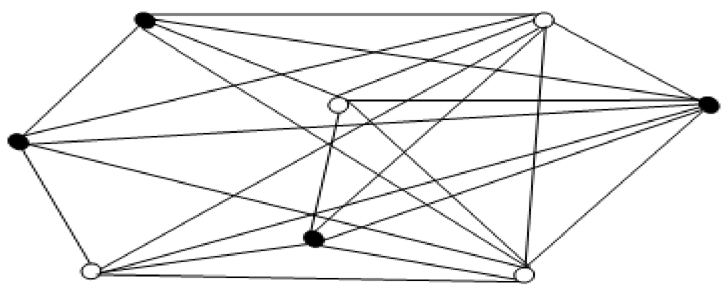
The join graph of any two cycle graphs C_n and C_m where n, m are both odd is a very cost effective graph.

Proof:

Let C_n and C_m be any two cycle graphs where n, m are both odd. Choose the cost effective bipartitions $\pi_1 = \{S_1, V_1 \setminus S_1\}$ and $\pi_2 = \{S_2, V_2 \setminus S_2\}$ such that $n(S_1) = n(V_1 \setminus S_1) + 1$ and $n(V_2 \setminus S_2) = n(S_2) + 1$. Now consider the join graph $C_n \vee C_m$ with the cost effective bipartition $\pi = \{S, V \setminus S\}$, where $S = S_1 \cup S_2$ and $V \setminus S = (V_1 \setminus S_1) \cup (V_2 \setminus S_2)$. Now, let $u \,\epsilon\, S_1$, if u is a very cost effective vertex, then u must be adjacent to more vertices of $V_1 \setminus S_1$ than in S_1 and since u is adjacent to all the vertices of S_2 and $V_2 \setminus S_2$ and $n(S_2) = n(V_2 \setminus S_2) - 1$, then u is still a very cost effective vertex in the join graph $C_n \vee C_m$. Now , suppose $u \,\epsilon\, S_1$ is a cost effective vertex , then u will be adjacent to at least as many vertices in $V_1 \setminus S_1$ as in S_1 and since u is adjacent to all the vertices of S_2 and $V_2 \setminus S_2$ and $n(V_2 \setminus S_2) = n(S_2) + 1$. Thus u is adjacent to more vertices in S than in $V \setminus S$ by at least one and consequently u is a very cost effective vertex. Now , let $u \,\epsilon\, (V_1 \setminus S_1)$, then u must be adjacent to more vertices in S_1 than in $V_1 \setminus S_1$ certainly by at least two and since u is adjacent to all the vertices of S_2 and $(V_2 \setminus S_2)$ and $n(S_2) = n(V_2 \setminus S_2) - 1$, then u is still a very cost effective vertex in the join graph $C_n \vee C_m$ .Also , we can prove that any vertex of S_2 or $V_2 \setminus S_2$ is a very cost effective vertex and hence π is a very cost bipartition , i.e., $C_n \vee C_m$ is a very cost effective graph.

Example (2.8)

The join graph $C_3 \vee C_5$ is a very cost effective, see Fig. 2.7.



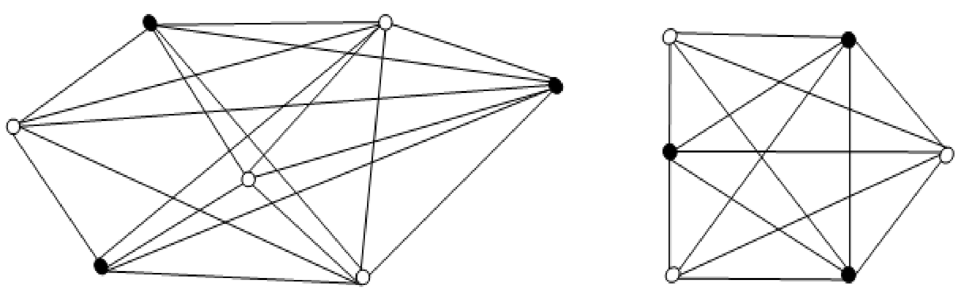
$C_3 \vee C_5$ *is very cost effective*

Fig. 2.7

Now let G_1 be a very cost effective graph and G_2 any non trivial connected graph. The join graph of $G_1 \vee G_2$ is not allways very cost effective, the following example illustrate this fact.

Example (2.9)

The join graph $C_3 \vee C_4$ shown in Fig. 2.8a is not a very cost effective graph while the join graph $P_3 \vee C_3$ is very cost effective, see Fig. 2.8b.



$C_3 \vee C_4$ *is not very cost effective* $P_3 \vee C_3$ *is very cost effective*

(a) (b)

Fig. 2.8

Thus the answer to the second question is, in general, no. The answer is yes in the following case.

Theorem (2.10)

The join graph of any path P_n and cycle C_n where n is odd is a very cost effective graph.

Proof:

By the same procedure as in theorem (2.7) we can prove this theorem.

Now, we come to the third question. The answer of the third question is also, in general, no, i.e., if G_1 and G_2 are two very cost effective graphs, then the join graph $G_1 \vee G_2$ may or may not be very cost effective . For example , if G_1 and G_2 are the very cost effective graphs shown in Fig. 2.9, then the join graph $G_1 \vee G_2$ is not very cost effective.

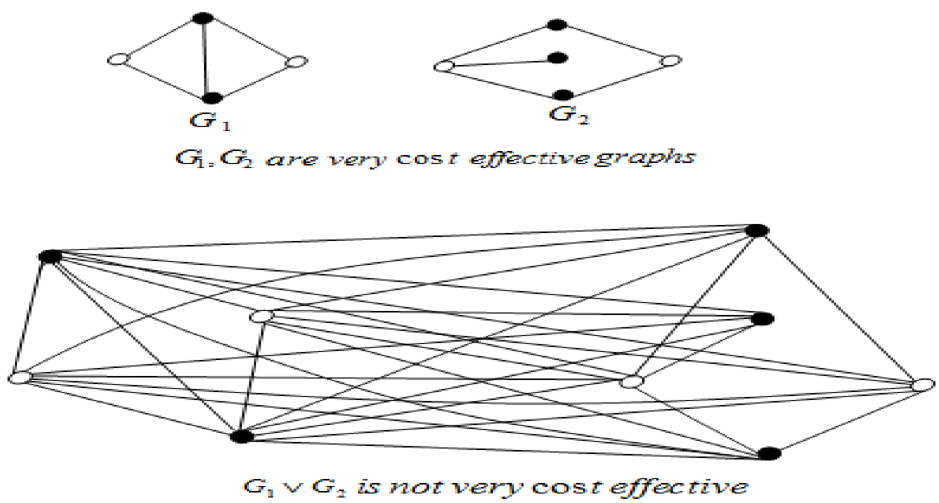


Fig. 2.9

While the join graph $K_4 \vee K_{2,2}$ is a very cost effective graph, see Fig. 2.10.

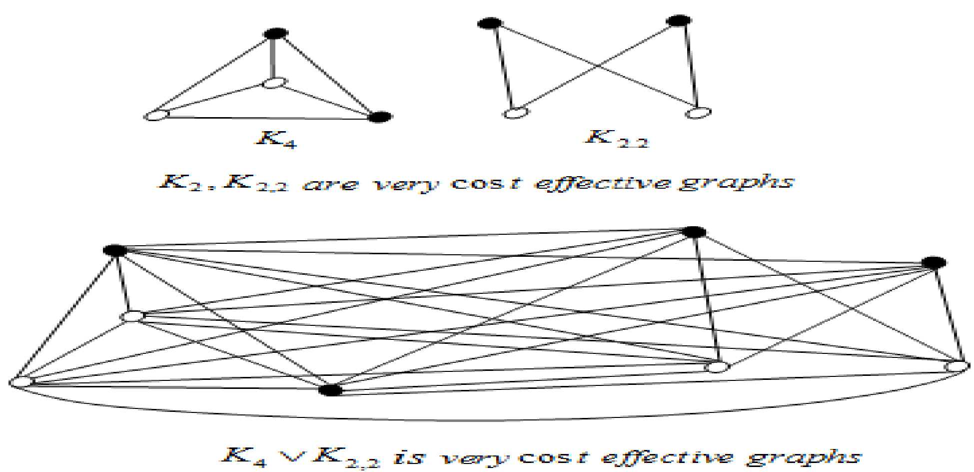


Fig. 2.10

The following theorem gives the condition for which the join graph of two very cost effective graphs is very cost effective.

Theorem (2.11)

The join graph of two non trivial connected very cost effective graphs G_1 and G_2 is a very cost effective graph if $n(G_1) + n(G_2)$ is even.

Proof:

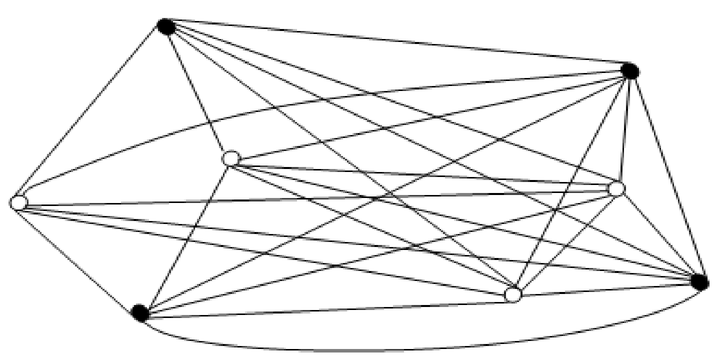
Let G_1 and G_2 be any non trivial connected graphs, we have two cases:

(1) $n(G_1)$ and $n(G_2)$ are both even. Then each graph has a very cost effective bipartition. Let $\pi_1 = \{S_1, V_1 \setminus S_1\}$ and $\pi_2 = \{S_2, V_2 \setminus S_2\}$, be such that $n(S_1) = n(V_1 \setminus S_1)$ and $n(S_2) = n(V_2 \setminus S_2)$. Now consider the join graph $G_1 \vee G_2$ with the cost bipartition $\pi = \{S, V \setminus S\}$, where $S = S_1 \cup S_2$ and $V \setminus S = (V_1 \setminus S_1) \cup (V_2 \setminus S_2)$. Let $u \in S_1$, then u is adjacent to more vertices of $V_1 \setminus S_1$ than in S_1 and since u is adjacent to all the vertices of S_2 and $V_2 \setminus S_2$, then u is still a very cost effective vertex in the join graph $G_1 \vee G_2$. Also we can prove that any vertex of $V_1 \setminus S_1$ or S_2 or $V_2 \setminus S_2$ is a very cost effective vertex and hence π is a very cost effective bipartition, i.e., $G_1 \vee G_2$ is a very cost effective graph.

(2) $n(G_1)$ and $n(G_2)$ are both odd. Suppose that $\pi_1 = \{S_1, V_1 \setminus S_1\}$ and $\pi_2 = \{S_2, V_2 \setminus S_2\}$ are very cost effective bipartitions such that $n(S_1) = n(V_1 \setminus S_1) + 1$ and $n(V_2 \setminus S_2) = n(S_2) + 1$. Let $\pi = \{S, V \setminus S\}$ be a cost bipartition of the join graph $G_1 \vee G_2$, where $S = S_1 \cup S_2$ and $V \setminus S = (V_1 \setminus S_1) \cup (V_2 \setminus S_2)$. Now, let $u \in S_1$, then u must be adjacent to more vertices of $V_1 \setminus S_1$ than in S_1 and since u is adjacent to all the vertices of S_2 and $V_2 \setminus S_2$ and $n(S_2) = n(V_2 \setminus S_2) - 1$, then u is still a very cost effective vertex in the join graph $G_1 \vee G_2$. Now, let $u \in V_1 \setminus S_1$, then u must be adjacent to more vertices in S_1 than in $V_1 \setminus S_1$ certainly by at least two and since u is adjacent to all the vertices of S_2 and $V_2 \setminus S_2$, then u is adjacent to more vertices in S than in $V \setminus S$ by at least one and u is a very cost effective vertex in the join graph $G_1 \vee G_2$. Also we can prove that any vertex of S_2 or $V_2 \setminus S_2$ is very cost effective vertex and hence π is a very cost effective bipartition, i.e., $G_1 \vee G_2$ is a very cost effective graph.

Examples (2.12)

(1) The join graph $C_4 \vee W_{1,3}$ of the very cost effective graphs C_4 and $W_{1,3}$ is once again a very cost effective graph, see Fig. 2.11.



$C_4 \vee W_{1,3}$ *is very cost effective*

Fig. 2.11

(2) Let G_1 and G_2 be the very cost effective graphs shown in Fig. 2.12. Then the join graph $G_1 \vee G_2$ is also very cost effective graph, see Fig. 2.12.

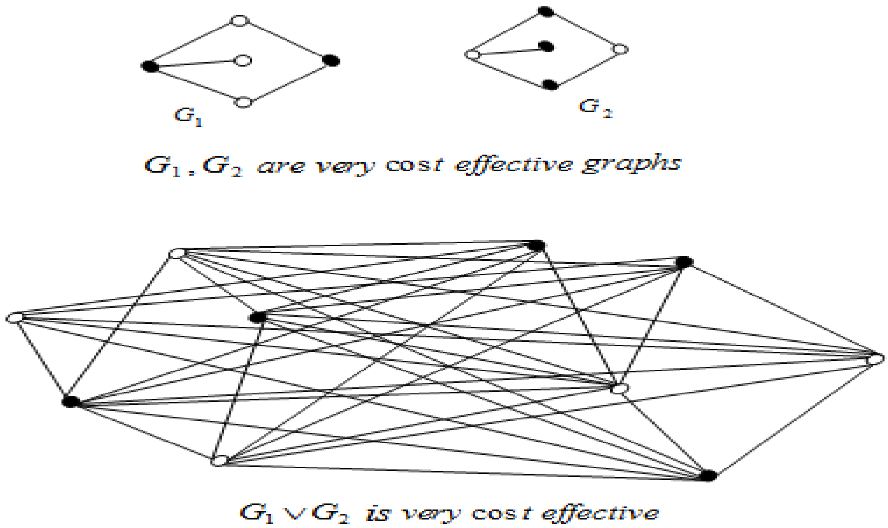


Fig. 2.12

3 The Join Graph Folding of Very Cost Effective Graphs

In this section we study very cost effective graph folding of a new graph obtained by the join of two other graphs.

Definition (3.1)

Let $G_1=(V_1 , E_1)$,$G_2=(V_2 , E_2)$, $G_3=(V_3 , E_3)$ and $G_4=(V_4 , E_4)$ be graphs. Let f: $G_1 \rightarrow G_3$ and g: $G_2 \rightarrow G_4$ be graph maps. A join map $f \vee g$: $G_1 \vee G_2 \rightarrow G_3 \vee G_4$ is a map defined by

(i) For each vertex $v \in V_1 \cup V_2$, $(f \vee g)(v)= \begin{cases} f(v), & \text{if } v \in V_1 \\ g(v), & \text{if } v \in V_2 \end{cases}$

(ii) For each edge $e =(v_1,v_2)$,$v_1 \in V_1$ and $v_2 \in V_2$, $(f \vee g)\{e\}= \{f(v_1),g(v_2)\} \in G_3 \vee G_4$.

(iii) If $e=(u_1 , v_1) \in E_1$, then $(f \vee g)\{e\}=\{f(u_1) , f(v_1)\}$, also if $e=(u_2 , v_2) \in E_2$, then $(f \vee g)\{e\}= (f \vee g)\{(u_2 , v_2)\} = \{ g(u_2) , g(v_2)\}$, [7].

Definition (3.2)

A graph folding f: $G_1 \rightarrow G_2$ between two graphs G_1 and G_2 is a very cost effective graph folding iff the image $f(G_1) = H \subseteq G_2$ is a very cost effective graph [8].

It should be noted that the image of a graph folding f: G → G of a very cost effective graph G may or may not be a very cost effective graph , e.g., if G is the cycle graph C_4 where $V(C_4)=\{ u_1, u_2, u_3 , u_4 \}$ and $E(C_4)=\{ e_1, e_2 , e_3 , e_4 \}$ then the graph folding f: $C_4 \rightarrow C_4$ defined by $f\{u_1 ,u_2 ,u_3 ,u_4\} = \{u_3 ,u_2 ,u_3 ,u_4\}$ and $f\{e_1 ,e_2 ,e_3 ,e_4\} = \{e_4 ,e_3 ,e_3 ,e_4\}$ is a very cost effective graph folding since the image is P_3 , see Fig. 3.1.

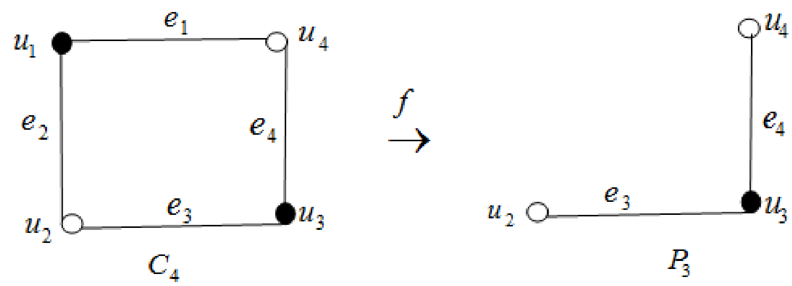


Fig. 3.1

From now on the omitted vertices or edges will be mapped onto themselves.

Also if G is the very cost effective graph shown in Fig. 3.2 where V(G)={ v_1, v_2, v_3, v_4} and E(G)={ e_1 , e_2 , e_3 ,e_4 ,e_5 } then the graph folding g: G → G defined by g{v_1 } = {v_3} and g{e_1 , e_2} = {e_4,e_3} is not a very cost effective graph folding.

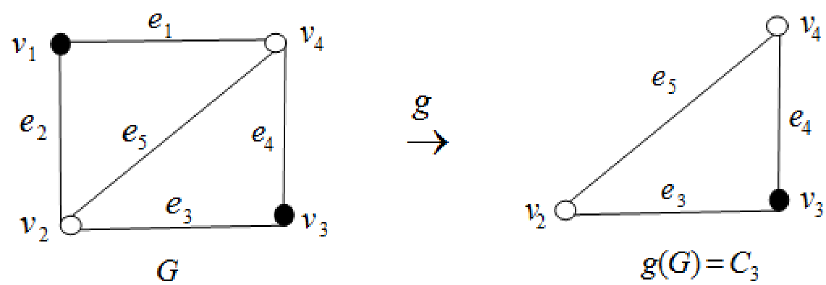


Fig. 3.2

Once again if f and g are both very cost effective graph foldings, the join map (f ∨ g) may or may not be a very cost effective graph folding. The following two examples illustrate this.

Examples (3.3)

(1) Let G_1 = P_3 and G_2 = P_4 as shown in Fig. 3.3. Let f : G_1 → G_1 and g: G_2 → G_2 be the very cost effective graph foldings defined by f{v_1 }={v_3}, f{e_1 }={e_2} and g{u_1 ,u_4 }={u_3 ,u_2} and g{e_3 ,e_5}={ e_4 ,e_4 }. The join graph folding f∨g: G_1∨ G_2 → G_1∨ G_2 defined by (f∨g){u_1 ,u_4 ,v_1 }={u_3 ,u_2 ,v_3} is a very cost effective graph folding.

Theorem (3.4)

Let G_1 and G_2 be any connected graphs and f, g are very cost effective graph foldings of G_1 and G_2, respectively. Then (f∨g)(G_1∨ G_2) is a very cost effective graph folding if no. V(f(G_1)) + no. V(g(G_2)) is even.

The proof is almost the same as the proof of theorem (2.11).

Example (3.5)

Consider the wheel graphs $W_{1,4}$ and $W_{1,6}$ shown in Fig. 3.4. Let f: $W_{1,4}$ → $W_{1,4}$ and g: $W_{1,6}$ → $W_{1,6}$ be the very cost effective graph foldings defined By f{u_1}={u_3} and f{e_1 , e_4 ,e_5}={e_2 ,e_3 , e_7} and g{v_1 ,v_5 , v_6 }={v_3

$,v_3$, $v_4\}$ and $g\{e_1, e_4, e_5, e_6, e_7, e_{11}, e_{12}\}=\{e_2, e_3, e_4, e_3, e_9, e_9, e_{10}\}$. Then The join graph folding $f \vee g : W_{1,4} \vee W_{1,6} \to W_{1,4} \vee W_{1,6}$ is a very cost effective graph folding, see Fig. 3.4.

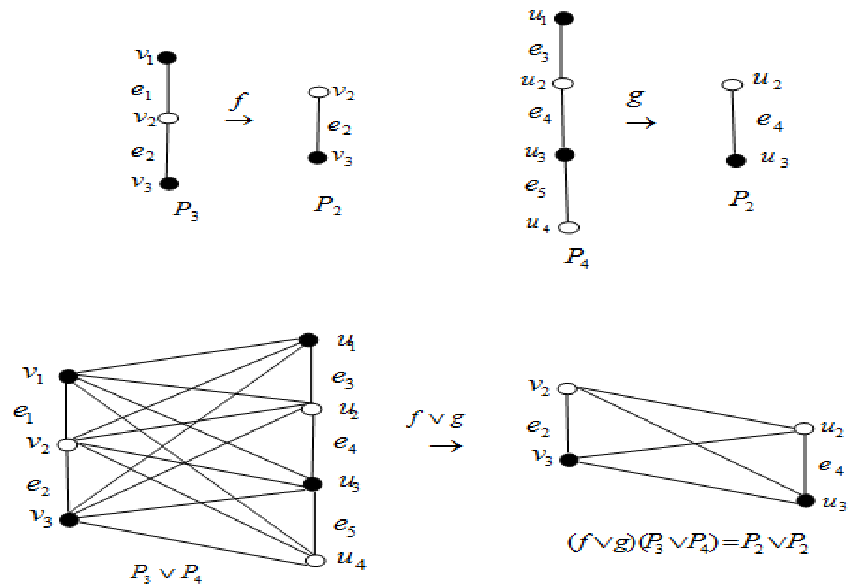


Fig. 3.3

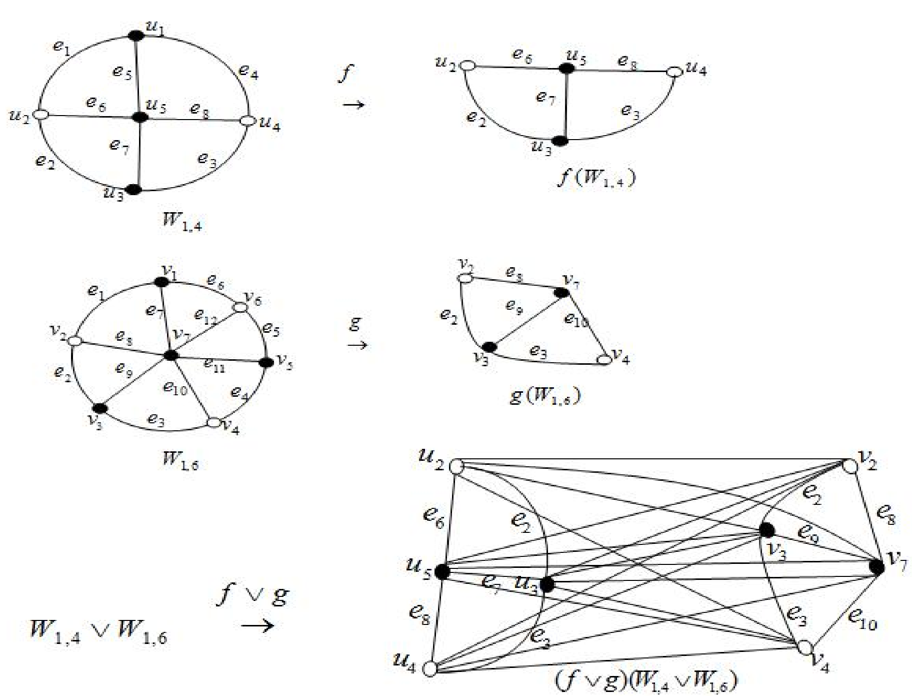


Fig. 3.4

Competing Interests

Authors have declared that no competing interests exist.

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