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*Review*

# Some propertıes of fuzzy contınuıty functıons

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**In this study, some definitions and features related to fuzzy continuity, fuzzy membership functions and fuzzy continuous functions are examined. Using these definitions and properties, some theorems about fuzzy continuous functions have been proved.**

**Key words:** Fuzzy continuity, fuzzy function, fuzzy set.

## INTRODUCTİON

In 1965, Zadeh's fuzzy set concept was the basis of mathematical testing of the fuzzy concept that exists in our real world and the formation of new branches in mathematics. The fuzzy set concept corresponding to unexplained physical situations gives useful applications on many topics such as statistics, data processing and linguistics. A lot of research has been done on this subject since 1965. It has been investigated whether or not the concepts and the theorems in the classical theory of mathematics can be applied to the fuzzy theory. After publishing the fuzzy clusters of Zadeh in 1965 and defining Chang's fuzzy topological space in 1968, many concepts in general topology have been moved to fuzzy topological spaces and original studies have begun to be obtained. It is also known that various functional types play an important role in classical topology. Many researchers have extended fuzzy topology to such studies.Various concepts such as fuzzy topological spaces, fuzzy uniform spaces, fuzzy groups, fuzzy vector spaces, fuzzy measures and fuzzy integrals have been investigated by using fuzzy set concept. In addition, Azad (1981) has studied fuzzy semi continuity, fuzzy almost continuity and fuzzy weak continuity. Yalvaç (1987) introduced the concepts on fuzzy sets and function on fuzzy spaces. Similar to the minimal structure in topological spaces in Alimohammady and Roohi (2006), they defined fuzzy minimal space concept and developed many theorems and features. In fuzzy minimal spaces, fuzzy minimal functions are defined and some continuity types are given to investigate the relation between them. Dealing with uncertainties is a major problem in many areas such as engineering, medical science, environmental science, social science, etc. These kinds of problems cannot be dealt with by classical methods, because classical methods have inherent difficulties. To overcome these kinds of difficulties, Molodtsov (1990) proposed a completely new approach, which is called soft set theory, for modeling uncertainty. Maji et al. (2003) studied the theory of soft sets and developed several basic nations of soft sets theory in 2003 (Uluçay et al., 2016).

Continuity is a fact known to have an important place in fuzzy topological space as well as in general topology. Like many concepts in general topology, the notion of continuity has moved to fuzzy topological spaces and many original works have begun to be obtained. In the

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literature survey, it has been observed that there are many studies on continuity and continuity types in fuzzy topological spaces.

In this study, concepts of fuzzy set and fuzzy continuity are given and some basic features of them are examined. Some of the features of fuzzy set and fuzzy continuous functions have been proved by taking advantage of these basic features examined.

## SOME BASIC DEFINITIONS

**Definition 1**

Let two fuzzy sets of X be A and B, and let membership function of A and B fuzzy subjets be repsented by $\mu_A(x)$ and $\mu_B(x)$ respectively, for $x \in X$. If membership degrees of membership functions of A and B in each point of X are equal to each other, then it means that A and B fuzzy sets are equal and written as;

$$A = B \Leftrightarrow \mu_A(x) = \mu_B(x), \text{ for all } x \in X$$

**Definition 2**

It ıs defined as $\mu_A(x) \leq \mu_B(x)$ for all $x \in X$, where any two of the fuzzy sets are A and B. A means subjet of B and written as $A \subset B$ . Briefly defined as:

$$A \subset B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \text{ for all } x \in X$$

**Definition 3**

Let A and B be any two fuzzy sets in X. Then we define:

$$C = A \cup B \Leftrightarrow \mu_C(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}, \text{ for all } x \in X.$$

More generally, for a family of fuzzy sets, $A = \{A_i \, ; \, i \in I\}$, the union, $C = \bigcup_{i \in I} A_i$ is defined by:

$$\mu_C(x) = \text{Sup}\{\mu_{A_i}(x)\}, \text{ for all } x \in X.$$

**Definition 4**

Let A and B be any two fuzzy sets in X. Intersect of these fuzzy sets is defined to be membership function as follows:

$$D = A \cap B \Leftrightarrow \mu_D(x) = \text{Max}\{\mu_A(x), \mu_B(x)\}, \text{ for all } x \in X.$$

More generally, for a family of fuzz sets, $A = \{A_i \, ; \, i \in I\}$, the intersection, $D = \bigcap_{i \in I} A_i$ is defined by:

$$\mu_D(x) = \text{Inf}\{\mu_{A_i}(x)\}, \text{ for all } x \in X.$$

**Definition 5**

It means that the necassary and sufficient condition of membership function are identically zero on X.

The symbol $\varnothing$ used to denote the empty fuzzy set ($\mu_\varnothing(x) = 0$ for all $x \in X$ ) and for X we have the definition $\mu_X(x) = 1$ for all $x \in X$.

**Definition 6**

Complement of A fuzzy set is represented by A' and defined by membership function of $\mu_{A'}(x) = 1 - \mu_A(x)$ for all $x \in X$.

## F- CONTİNUOUS FUNCTİONS

**Definition 7**

Let f be function from X to Y . Let B be a fuzzy set in Y with membership function $\mu_B(y)$. Then the inverse of B, written as $f^{-1}(B)$, is fuzzy set in X whose membership function is defined by $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$, for all $x \in X$.

Conversely, let A be a fuzzy set in X with membership function $\mu_A(x)$. The image of A, written as f [A], is a fuzzy set in Y whose membership function is given by:

$$\mu_{f[A]}(y) = \sup_{z \in f^{-1}(y)} \{\mu_A(z)\}, \; f^{-1}[y] \neq \varnothing$$

$= 0$ otherwise

for all y in Y , where $f^{-1}[y] = \{X : f(x) = y)\}$.

**Theorem 1**

Let f be a function from X to Y, and I be not empty any index set. The folowing statements are true:

**1.** If $A_i \subset X$ for every $i \in I$ then $f\left(\bigcup_{i \in I} A_i\right) = \bigcup_{i \in I} f(A_i)$

**2.** If $B_i \subset Y$ for every $i \in I$ then $f^{-1}\left(\bigcup_{i \in I} B_i\right) = \bigcup_{i \in I} f^{-1}(B_i)$

**3.** If $A, B \subset X$ then $f(A \cap B) \subset f(A) \cap f(B)$

**4.** If $B_i \subset Y$ for every $i \in I$ then $f^{-1}\left(\bigcap_{i \in I} B_i\right) = \bigcap_{i \in I} f^{-1}(B_i)$

***Proof 1***

For each $y \in Y$ , if $f^{-1}(y)$ is not empty , then:

$$\mu_{f(\bigcup_{i \in I} A_i)}(y) = \sup_{x \in f^{-1}(y)} \{\mu_{\bigcup_{i \in I} A_i}(x)\} = \sup_{x \in f^{-1}(y)} \{ \sup_{i \in I} \{ \mu$$

$_{A_i}(x)\}\}$, from Definition 7

$\mu_{f(\bigcup_{i\in I} A_i)}(y) = \sup_{i\in I}\{\sup_{x\in f^{-1}(y)}\{\mu_{A_i}(x)\}$, from Definition 7

$\mu_{f(\bigcup_{i\in I} A_i)}(y) = \sup_{i\in I}\{\mu_{f(A_i)}(y)\}$, Hence we have written:

$\mu_{f(\bigcup_{i\in I} A_i)}(y) = \mu_{\bigcup_{i\in I} f(A_i)}(y)$. This shows that $f(\bigcup_{i\in I} A_i) = \bigcup_{i\in I} f(A_i)$.

**Proof 2**

For all $x\in X$

$\mu_{f^{-1}(\bigcup_{i\in I} B_i)}(x) = \mu_{\bigcup_{i\in I} B_i}(f(x))$, from Definition 7

$= \sup_{i\in I}\{\mu_{B_i}(f(x))\}$

$= \sup_{i\in I}\{\mu_{f^{-1}(B_i)}(x)\}$. Hence for $x\in X$ we have written

$\mu_{f^{-1}(\bigcup_{i\in I} B_i)}(x) = \mu_{\bigcup_{i\in I} f^{-1}(B_i)}(x)$. This sows that $f^{-1}(\bigcup_{i\in I} B_i) = \bigcup_{i\in I} f^{-1}(B_i)$

***Proof 3***

Since $A \cap B \subset A$ and $A \cap B \subset B$ for all $x\in X$, $\mu_{A\cap B}(x) \le \mu_A(x)$ and $\mu_{A\cap B}(x) \le \mu_B(x)$ we have written. Hence for all $x\in X$ $\mu_{f(A\cap B)}(x) \le \mu_{f(A)}(x)$ and $\mu_{f(A\cap B)}(x) \le \mu_{f(B)}(x)$.

Similarly, for all $y\in Y$ $\mu_{f(A\cap B)}(y) \le \mu_{f(A)}(y)$ and $\mu_{f(A\cap B)}(y) \le \mu_{f(B)}(y)$. Hence, for all $y\in Y$, $\mu_{f(A\cap B)}(y) \le \text{Min}\{\mu_{f(A)}(y), \mu_{f(B)}(y)\} = \mu_{f(A)\cap f(B)}(y)$. This shows that $f(A \cap B) \subset f(A) \cap f(B)$.

***Proof 4***

For all $x\in X$

$\mu_{f^{-1}(\bigcap_{i\in I} B_i)}(x) = \mu_{\bigcap_{i\in I} B_i}(f(x)) = \inf_{i\in I}\{\mu_{B_i}(f(x))\}$, from Definition 7

$= \inf_{i\in I}\{\mu_{f^{-1}(B_i)}(x)\}$. Hence, we have written

$\mu_{f^{-1}(\bigcap_{i\in I} B_i)}(x) = \mu_{\bigcap_{i\in I} f^{-1}(B_i)}(x)$. This shows that $f^{-1}(\bigcap_{i\in I} B_i) = \bigcap_{i\in I} f^{-1}(B_i)$.

**Theorem 2**

Let f be a function from X to Y. The following statements are true:

1. If $B \subset Y$ then $f^{-1}(B') = f^{-1}(B)'$
2. If $A \subset X$ then $f(A)' = f(A')$
3. If $A_1, A_2 \subset X$ and $A_1 \subset A_2$ then $f(A_1) \subset f(A_2)$
4. If $B_1, B_2 \subset Y$ and $B_1 \subset B_2$ then $f^{-1}(B_1) \subset f^{-1}(B_2)$
5. If $A \subset X$ then $A \subset f^{-1}(f(A))$
6. If $B \subset Y$ then $f(f^{-1}(B)) \subset B$
7. If f one –to one and $A \subset X$ then $f^{-1}(f(A)) = A$

**Definition 8**

A fuzzy topoloji is a family F of fuzzy sets in X which satisfies the following contitions:

1. Ø ve X є F
2. If A , B є F, then $A \cap B$ є F
3. If $A_i$ є F for each iєI, then $\bigcup A_i$ є F.

F is called fuzzy topology for X, and the pair ( X, F) is a fuzzy topological space every member of F is called a F-open fuzzy sets. A fuzzy set is F-closed if its complement is F- open.

**Definition 9**

A function from a fuzzy topolojical space $(X, F_1)$ to a fuzzy topological space $(Y, F_2)$ is F- Continuous if the inverse of each $F_2$-open fuzzy set is $F_1$-open.

**Theorem 3**

If $f : X \to Y$ and $g : Y \to Z$ are two F–continuous functions $(gof) : X \to Z$ function is also F–continuous.

**Theorem 4**

It is accepted that $(X, F_1)$ and $(Y, F_2)$ are two fuzzy topological space, and $f : X\to Y$ function is given. The necessary and sufficient condition for being F-continuous of f function is X must be fuzzy closed subset for each B fuzzy closed subset in Y.

***Proof 5***

It was accepted that X and Y were two fuzzy topological space and $f : X \to Y$ was a F–continuous function. In this case we will show that reverse image of each B fuzzy closed subset obtained from Y fuzzy topological space is fuzzy closed subset in X.

It was accepted that f function was F-continuous and B was any fuzzy closed subset taken from Y. Also X is fuzzy open subset because of the Theorem 2 and fuzzy continuous of f, $f^{-1}(B') = \{f^{-1}(B)\}'$. In other words reverse image's complement of B fuzzy closed subset is fuzzy open subset of X. So, $f^{-1}(B)$ is fuzzy closed subset of X.

This implies that we accept that reverse image of every fuzzy closed subset obtained from Y is fuzzy closed subset of X. In this case it will be shown that f is F-continuous. So B was a fuzzy open set which was chosen in Y arbitrary. The complement of this set, B' is to be a fuzzy closed set in Y. $f^{-1}(B') = f^{-1}(B)'$ will be fuzzy closed set in X because of the Theorem 2. That is, $f^{-1}(B)$ will be fuzzy open set in X. Consequently, it is seen from Definition 3 that f is F-continuous.

## Conclusion

In this study, some definitions of fuzzy membership functions and fuzzy continuity concepts defined by Chang (1968) were investigated by Azad (1981). In Yalvaç's (1987) work, only the expressions given in Theorem 1 have been proven. Furthermore, Yalvaç (1987) has also proven that the Theorem 4 for fuzzy continuous functions has been established and utilized without any stress.

## CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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